

Introducción a las Ecuaciones diferenciales Ordinarias (EDOs III)

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- El avance se realiza paso a paso en el tiempo pero se realiza un proceso iterativo para dar por finalizado el paso.
- Métodos más difundidos:
 - Euler implícito
 - Euler-Gauss

Algoritmo: $\underline{y_{i+1}} = y_i + f(\underline{x_{i+1}}, \underline{y_{i+1}}) \Delta x$



Se debe iterar hasta cerrar la igualdad

Ejemplo: $\frac{dy}{dx} = x - \sqrt{y} \quad x \in [0, 3] \quad y(0) = 100 \quad \Delta x = 0.2$

1er punto:

$$y_1 = y_0 + f(x_1, y_1) \Delta x$$

$$y_1 = y_0 + (x_1 - \sqrt{y_1}) \Delta x$$

$$y_1 = 100 + (0.2 - \sqrt{y_1}) 0.2$$

Ecuación no lineal

Aprox. Suc.
Wegstein
Newton
Secante

$$y_1 = 100 + \left(0.2 - \sqrt{y_1}\right)0.2$$

$$y_1^* = 100 + \left(0 - \sqrt{100}\right)0.2 = 98 \quad \text{¡Euler Explícito!}$$

y_1	$F(y_1)$	error
98.0000		

$$\begin{cases} \frac{dy}{dx} = x - \sqrt{y} \\ x \in [0, 3] \quad y(0) = 100 \end{cases} \quad y_{i+1} = y_i + f(x_{i+1}, y_{i+1}) \Delta x$$

i	x_i	y_i	x_{i+1}	y_{i+1}^*	y'_{i+1}	error	$\Delta x = 0.2$
0	0	100.00000	0.20000	98.05950	-9.70250	0.00000	
1	0.2	98.05950	0.40000	96.17809	-9.40704	0.00000	
2	0.4	96.17809	0.60000	94.35536	-9.11367	0.00000	
3	0.6	94.35536	0.80000	92.59087	-8.82242	0.00000	
4	0.8	92.59087	1.00000	90.88421	-8.53332	0.00000	
5	1	90.88421	1.20000	89.23493	-8.24642	0.00000	
6	1.2	89.23493	1.40000	87.64257	-7.96176	0.00000	
7	1.4	87.64257	1.60000	86.10670	-7.67937	0.00000	
8	1.6	86.10670	1.80000	84.62684	-7.39928	0.00000	
9	1.8	84.62684	2.00000	83.20253	-7.12154	0.00000	
10	2	83.20253	2.20000	81.83330	-6.84618	0.00000	
11	2.2	81.83330	2.40000	80.51866	-6.57322	0.00000	
12	2.4	80.51866	2.60000	79.25811	-6.30270	0.00000	
13	2.6	79.25811	2.80000	78.05118	-6.03466	0.00000	
14	2.8	78.05118	3.00000	76.89736	-5.76911	0.00000	
15	3	76.89736	3.20000				

Algoritmo:

$$\underline{y_{i+1}} = y_i + \frac{\Delta x}{2} \left[f(x_i, y_i) + f(x_{i+1}, \underline{y_{i+1}}) \right]$$

Se debe iterar hasta cerrar la igualdad

Análisis:

$$y_{i+1} = \frac{y_i}{2} + \frac{y_i}{2} + \frac{\Delta x}{2} \left[f(x_i, y_i) + f(x_{i+1}, y_{i+1}) \right]$$

$$y_{i+1} = \frac{1}{2} \left[\boxed{y_i + f(x_i, y_i)} + \boxed{y_i + f(x_{i+1}, y_{i+1})} \right]$$

Euler explícito

Euler implícito

Algoritmo:
$$y_{i+1} = y_i + \frac{\Delta x}{2} \left[f(x_i, y_i) + f(x_{i+1}, y_{i+1}) \right]$$

Ejemplo: $\frac{dy}{dx} = x - \sqrt{y} \quad x \in [0, 3] \quad y(0) = 100 \quad \Delta x = 0.2$

1er punto:

$$y_1 = y_0 + \frac{\Delta x}{2} \left(f(x_0, y_0) + f(x_1, y_1) \right)$$

$$y_1 = y_0 + \frac{\Delta x}{2} \left(\left(x_0 - \sqrt{y_0} \right) + \left(x_1 - \sqrt{y_1} \right) \right)$$

$$y_1 = 100 + 0.1 \left(\left(0 - \sqrt{100} \right) + \left(0.2 - \sqrt{y_1} \right) \right)$$

Ecuación no lineal
Aprox. Suc.
Wegstein
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$$y_1 = 100 + 0.1 \left(\left(0 - \sqrt{100} \right) + \left(0.2 - \sqrt{y_1} \right) \right)$$

y_1	$F(y_1)$	error
98.0000	98.0300505	3.06638E-04
98.0300505	98.0298987	1.54816E-06
98.0298987	98.0298995	7.81821E-09
98.0298995	98.0298995	3.94819E-11

$$\begin{cases} \frac{dy}{dx} = x - \sqrt{y} \\ x \in [0, 3] \quad y(0) = 100 \end{cases} \quad y_{i+1} = y_i + \frac{\Delta x}{2} \left[f(x_i, y_i) + f(x_{i+1}, y_{i+1}) \right]$$

i	x _i	y _i	y' _i	x _{i+1}	y _{i+1} *	y' _{i+1}	error	Δx=0.2
0	0	100.00000	-10.00000	0.20000	98.02990	-9.70100	0.00000	
1	0.2	98.02990	-9.70100	0.40000	96.11939	-9.40405	0.00000	
2	0.4	96.11939	-9.40405	0.60000	94.26807	-9.10917	0.00000	
3	0.6	94.26807	-9.10917	0.80000	92.47551	-8.81642	0.00000	
4	0.8	92.47551	-8.81642	1.00000	90.74129	-8.52582	0.00000	
5	1	90.74129	-8.52582	1.20000	89.06496	-8.23742	0.00000	
6	1.2	89.06496	-8.23742	1.40000	87.44610	-7.95126	0.00000	
7	1.4	87.44610	-7.95126	1.60000	85.88423	-7.66737	0.00000	
8	1.6	85.88423	-7.66737	1.80000	84.37891	-7.38580	0.00000	
9	1.8	84.37891	-7.38580	2.00000	82.92968	-7.10657	0.00000	
10	2	82.92968	-7.10657	2.20000	81.53605	-6.82973	0.00000	
11	2.2	81.53605	-6.82973	2.40000	80.19754	-6.55531	0.00000	
12	2.4	80.19754	-6.55531	2.60000	78.91368	-6.28334	0.00000	
13	2.6	78.91368	-6.28334	2.80000	77.68396	-6.01385	0.00000	
14	2.8	77.68396	-6.01385	3.00000	76.50789	-5.74688	0.00000	
15	3	76.50789	-5.74688	3.20000				