Introducción a las Ecuaciones diferenciales Ordinarias (EDOs II)

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- Una manera de reducir el error con el método de Euler sería incluir términos de orden superior en la expansión de la serie de Taylor para la solución. Por ejemplo, al incluir el término de segundo orden

$$y(x) = y(x_0) + y'(x_0)(x - x_0) + \frac{1}{2} y''(x_0)(x - x_0)^2$$

$$y(x_1) = y(x_0) + y'(x_0)(x_1 - x_0) + \frac{1}{2} y''(x_0)(x_1 - x_0)^2$$

$$y(x_1) = y(x_0) + y'(x_0)\Delta x + \frac{1}{2} y''(x_0)\Delta x^2$$

$y' = f(x, y)$ EDO

$$y(x_1) = y(x_0) + f(x_0, y_0)\Delta x + \frac{1}{2} f'(x_0, y_0)\Delta x^2$$

$$y_{i+1} = y_i + f\left(x_i, y_i\right)\Delta x + \frac{1}{2} f^{'}\left(x_i, y_i\right)\Delta x^2$$

- Aunque la incorporación de términos de orden superior es simple para implementarse en los polinomios, su inclusión no es tan trivial cuando la EDO es más complicada.
- Las EDOs que están en función tanto de la variable dependiente como de la independiente requieren de la derivación usando la regla de la cadena.

$$f^{'}\left(x, y\right) = \frac{\partial f\left(x, y\right)}{\partial x} + \frac{\partial f\left(x, y\right)}{\partial y}\frac{dy}{dx}$$

$$f^{"}\left(x, y\right) = \frac{\partial\left[\frac{\partial f\left(x, y\right)}{\partial x} + \frac{\partial f\left(x, y\right)}{\partial y}\frac{dy}{dx}\right]}{\partial x} + \frac{\partial\left[\frac{\partial f\left(x, y\right)}{\partial x} + \frac{\partial f\left(x, y\right)}{\partial y}\frac{dy}{dx}\right]}{\partial y}\frac{dy}{dx}$$

Ejercicio: Taylor 2do orden

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$$\begin{cases} \dfrac{dy}{dx} = x - \sqrt{y} \\ x \in [0,3] \quad y(0) = 100 \end{cases}$$

$$y_{i+1} = y_i + f(x_i, y_i)\Delta x + \frac{1}{2} f'(x_i, y_i)\Delta x^2$$

i	x	y	y'	y''	Δx= 0.2
0	**0**	**100.00000**	-10.00000	1.50000	
1	0.2	98.03000	-9.70101	1.48990	
2	0.4	96.11960	-9.40406	1.47960	
3	0.6	94.26838	-9.10919	1.46910	
4	0.8	92.47592	-8.81644	1.45840	
5	1	90.74180	-8.52585	1.44751	
6	1.2	89.06558	-8.23746	1.43642	
7	1.4	87.44682	-7.95130	1.42514	
8	1.6	85.88506	-7.66742	1.41368	
9	1.8	84.37985	-7.38585	1.40202	
10	2	82.93072	-7.10663	1.39019	
11	2.2	81.53720	-6.82980	1.37818	
12	2.4	80.19880	-6.55538	1.36600	
13	2.6	78.91505	-6.28341	1.35366	
14	2.8	77.68544	-6.01393	1.34116	
15	3	76.50947	-5.74697	1.32851	

$$f(x,y) = x - \sqrt{y}$$

$$f'(x,y) = \frac{\partial f(x,y)}{\partial x} + \frac{\partial f(x,y)}{\partial y}\frac{dy}{dx}$$

$$f'(x,y) = 1 - \frac{1}{2\sqrt{y}}y'$$

Método de Runge-Kutta

- Los métodos Runge-Kutta introducen varios parámetros a determinar, y hacen un promedio pesado de la función *f(x,y)* evaluada en diferentes puntos.
- Son más precisos que los métodos de Euler.
- En general los métodos Runge – Kutta tienen algoritmos de la forma:

$$y_{i+1} = y_i + \underbrace{\varnothing}_{\text{Incremento}} \Delta x$$

- Según se defina el incremento tenemos R-K de 2º, 3º o 4º orden.

Método de Runge-Kutta

- Dada la siguiente EDO: $y' = f(x, y)$
- Runge-Kutta de 2do orden corresponde a:

$$y_{i+1} = y_i + \varnothing \Delta x$$

Donde:

$$\varnothing = k_2$$

$$k_1 = f(x_i, y_i)$$

$$k_2 = f(x_{i+0.5}, y_i + 0.5 \Delta x k_1)$$

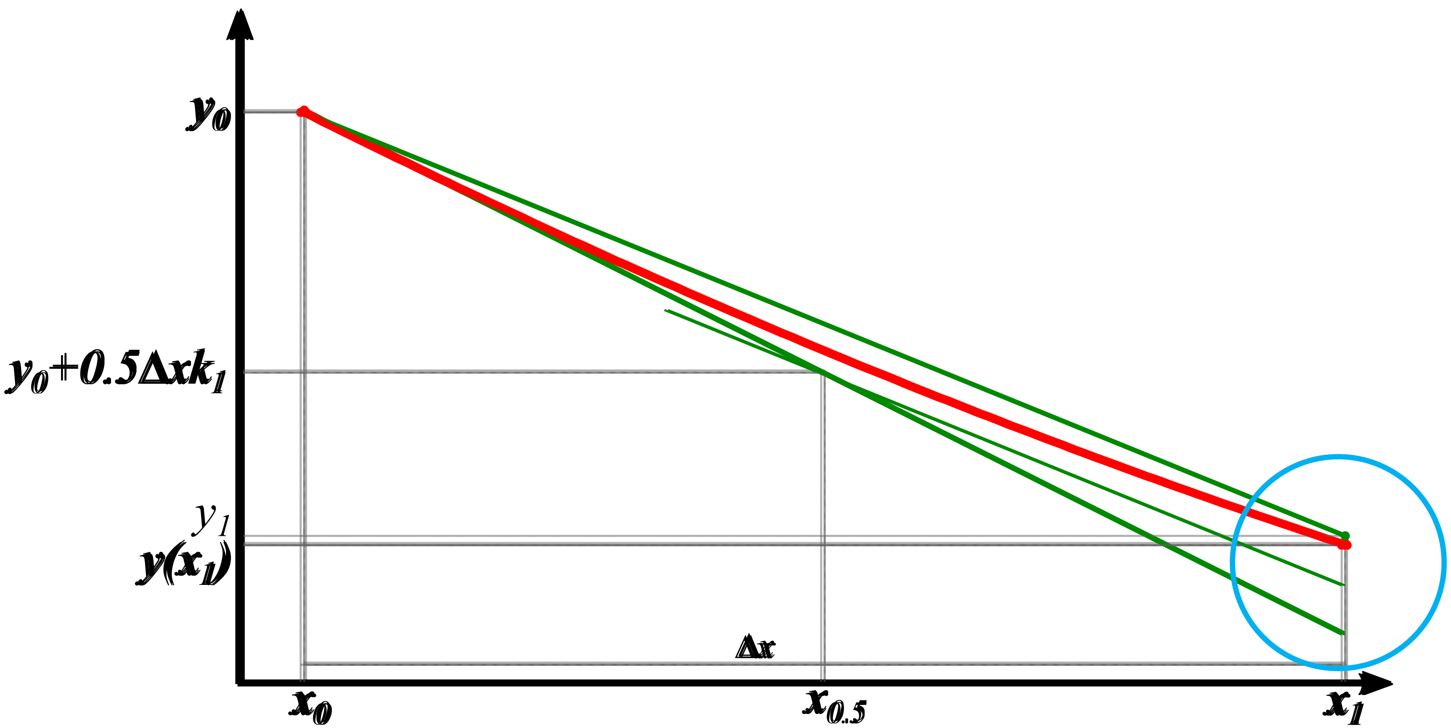
Método de Runge-Kutta

$$\varnothing = k_2$$

$$y_{i+1} = y_i + \varnothing \Delta x$$

$$k_1 = f\left(x_i, y_i\right)$$

$$k_2 = f\left(x_{i+0.5}, y_i + 0.5\Delta x k_1\right)$$



Ejercicio: RK 2

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$$\begin{cases} \dfrac{dy}{dx} = x - \sqrt{y} \\ x \in [0,3] \quad y(0) = 100 \end{cases}$$

$$y_{i+1} = y_i + \varnothing \Delta x$$

$$\varnothing = k_2$$

$$k_1 = f(x_i, y_i)$$

$$k_2 = f\left(x_{i+0.5}, y_i + 0.5\Delta x k_1\right)$$

$\Delta x = 0.2$

i	x	y	k1	$x_{i+0.5}$	$y_i + 0.5\Delta x k1$	k2
0	**0**	**100**	-10.00000	0.10000	99.00000	-9.84987
1	0.2	98.03003	-9.70101	0.30000	97.05992	-9.55190
2	0.4	96.11965	-9.40406	0.50000	95.17924	-9.25598
3	0.6	94.26845	-9.10919	0.70000	93.35753	-8.96217
4	0.8	92.47601	-8.81644	0.90000	91.59437	-8.67049
5	1	90.74192	-8.52586	1.10000	89.88933	-8.38100
6	1.2	89.06572	-8.23746	1.30000	88.24197	-8.09372
7	1.4	87.44697	-7.95131	1.50000	86.65184	-7.80870
8	1.6	85.88523	-7.66743	1.70000	85.11849	-7.52597
9	1.8	84.38004	-7.38586	1.90000	83.64145	-7.24557
10	2	82.93092	-7.10664	2.10000	82.22026	-6.96754
11	2.2	81.53742	-6.82981	2.30000	80.85444	-6.69191
12	2.4	80.19903	-6.55539	2.50000	79.54350	-6.41872
13	2.6	78.91529	-6.28343	2.70000	78.28695	-6.14799
14	2.8	77.68569	-6.01395	2.90000	77.08430	-5.87977
15	3	76.50974	-5.74698	3.10000	75.93504	-5.61407

Método de Runge-Kutta

- Runge-Kutta de 3er orden corresponde a:

$$y_{i+1} = y_i + \varnothing \Delta x$$

Donde: $\varnothing = \frac{1}{6}\left(k_1 + 4k_2 + k_3\right)$

$$k_1 = f\left(x_i, y_i\right)$$

$$k_2 = f\left(x_{i+0.5}, y_i + 0.5\Delta x k_1\right)$$

$$k_3 = f\left(x_{i+1}, y_i + 2\Delta x k_2 - \Delta x k_1\right)$$

Ejercicio: RK 3

$$\begin{cases} \dfrac{dy}{dx} = x - \sqrt{y} \\ x \in [0,3] \quad y(0) = 100 \end{cases}$$

i	x	y	k1	$x_{i+0.5}$	$y_i + 0.5\Delta x k1$	k2	x_{i+1}	$y_i + 2\Delta x k2 - \Delta x k1$	k3	$\Delta x = 0.2$
0	0	100	-10.00000	0.10000	99.00000	-9.84987	0.20000	98.06005	-9.70253	
1	0.2	98.02993	-9.70101	0.30000	97.05983	-9.55189	0.40000	96.14938	-9.40558	
2	0.4	96.11946	-9.40405	0.50000	95.17906	-9.25598	0.60000	94.29788	-9.11071	
3	0.6	94.26817	-9.10918	0.70000	93.35725	-8.96216	0.80000	92.50515	-8.81796	
4	0.8	92.47565	-8.81643	0.90000	91.59400	-8.67048	1.00000	90.77074	-8.52737	
5	1	90.74146	-8.52583	1.10000	89.88887	-8.38097	1.20000	89.09423	-8.23897	
6	1.2	89.06517	-8.23743	1.30000	88.24142	-8.09369	1.40000	87.47518	-7.95282	
7	1.4	87.44633	-7.95127	1.50000	86.65120	-7.80866	1.60000	85.91312	-7.66893	
8	1.6	85.88450	-7.66739	1.70000	85.11776	-7.52593	1.80000	84.40761	-7.38736	
9	1.8	84.37922	-7.38582	1.90000	83.64064	-7.24553	2.00000	82.95817	-7.10814	
10	2	82.93002	-7.10659	2.10000	82.21936	-6.96749	2.20000	81.56434	-6.83130	
11	2.2	81.53642	-6.82975	2.30000	80.85345	-6.69185	2.40000	80.22563	-6.55688	
12	2.4	80.19796	-6.55533	2.50000	79.54242	-6.41866	2.60000	78.94156	-6.28491	
13	2.6	78.91413	-6.28336	2.70000	78.28579	-6.14793	2.80000	77.71163	-6.01542	
14	2.8	77.68444	-6.01388	2.90000	77.08306	-5.87970	3.00000	76.53534	-5.74845	
15	3	76.50841	-5.74691	3.10000	75.93372	-5.61400	3.20000	75.41219	-5.48402	

Método de Runge-Kutta

- Runge-Kutta de 4to orden (mas difundido) corresponde a:

$$\varnothing = \frac{1}{6}\left(k_1 + 2k_2 + 2k_3 + k_4\right)$$

Donde:

$$k_1 = f\left(x_i, y_i\right)$$

$$k_2 = f\left(x_{i+0.5}, y_i + 0.5\Delta x k_1\right)$$

$$k_3 = f\left(x_{i+0.5}, y_i + 0.5\Delta x k_2\right)$$

$$k_4 = f\left(x_{i+1}, y_i + \Delta x k_3\right)$$

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Ejercicio: RK 4

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$$\begin{cases} \dfrac{dy}{dx} = x - \sqrt{y} \\ x \in [0,3] \quad y(0) = 100 \end{cases}$$

i	x	y	k1	$x_{i+0.5}$	$y_i + 0.5\Delta x k1$	k2	$y_i + 0.5\Delta x k2$	k3	x_{i+1}	$y_i + \Delta x k3$	k4	$\Delta x =$ 0.2
0	**0**	**100.00000**	-10.00000	0.10000	99.00000	-9.84987	99.01501	-9.85063	0.20000	98.02987	-9.70100	
1	0.2	98.02993	-9.70101	0.30000	97.05983	-9.55189	97.07474	-9.55265	0.40000	96.11940	-9.40405	
2	0.4	96.11946	-9.40405	0.50000	95.17906	-9.25598	95.19386	-9.25673	0.60000	94.26811	-9.10918	
3	0.6	94.26817	-9.10918	0.70000	93.35726	-8.96216	93.37196	-8.96292	0.80000	92.47559	-8.81642	
4	0.8	92.47565	-8.81643	0.90000	91.59401	-8.67048	91.60860	-8.67124	1.00000	90.74140	-8.52583	
5	1	90.74146	-8.52583	1.10000	89.88888	-8.38097	89.90336	-8.38174	1.20000	89.06511	-8.23743	
6	1.2	89.06517	-8.23743	1.30000	88.24143	-8.09369	88.25580	-8.09446	1.40000	87.44628	-7.95127	
7	1.4	87.44634	-7.95127	1.50000	86.65121	-7.80866	86.66547	-7.80943	1.60000	85.88445	-7.66739	
8	1.6	85.88451	-7.66739	1.70000	85.11777	-7.52593	85.13191	-7.52670	1.80000	84.37917	-7.38581	
9	1.8	84.37923	-7.38582	1.90000	83.64064	-7.24553	83.65467	-7.24629	2.00000	82.92997	-7.10659	
10	2	82.93002	-7.10659	2.10000	82.21937	-6.96749	82.23328	-6.96826	2.20000	81.53637	-6.82975	
11	2.2	81.53643	-6.82975	2.30000	80.85345	-6.69185	80.86724	-6.69262	2.40000	80.19791	-6.55533	
12	2.4	80.19796	-6.55533	2.50000	79.54243	-6.41866	79.55610	-6.41942	2.60000	78.91408	-6.28336	
13	2.6	78.91413	-6.28336	2.70000	78.28580	-6.14793	78.29934	-6.14869	2.80000	77.68440	-6.01388	
14	2.8	77.68445	-6.01388	2.90000	77.08306	-5.87970	77.09648	-5.88046	3.00000	76.50836	-5.74691	
15	3	76.50841	-5.74691	3.10000	75.93372	-5.61400	75.94702	-5.61476	3.20000	75.38546	-5.48248	