Modelo no-lineal

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17.12 An investigator has reported the data tabulated below for an experiment to determine the growth rate of bacteria k (per d), as a function of oxygen concentration c (mg/L). It is known that such data can be modeled by the following equation:

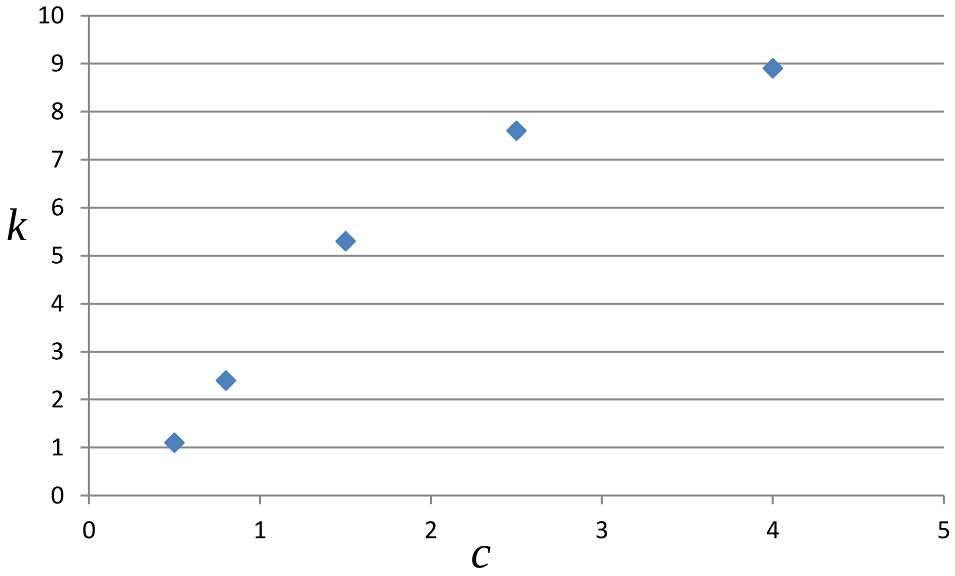
$$k = \frac{k_{\max} c^2}{c_s + c^2}$$

where c_s and $k_{\max}$ are parameters. Use a transformation to linearize this equation. Then use linear regression to estimate c_s and $k_{\max}$ and predict the growth rate at $c = 2$ mg/L.

c	0.5	0.8	1.5	2.5	4
k	1.1	2.4	5.3	7.6	8.9

Fuente:
S. Chapra and R. Canale, *Numerical Methods for Engineers, Sixth Edition*, 6 edition. Boston: McGraw-Hill Science/Engineering/Math, 2009.

c	k
0.5	1.1
0.8	2.4
1.5	5.3
2.5	7.6
4	8.9



Modelo propuesto: $$k = \frac{k_{max} c^2}{c_s + c^2}$$

¿Parámetros? → $k_{max} ; c_s$

¿Puedo expresar el residuo de manera lineal (r=Ax-y)? → NO

Se lo debe transformar en un modelo lineal en los parámetros.

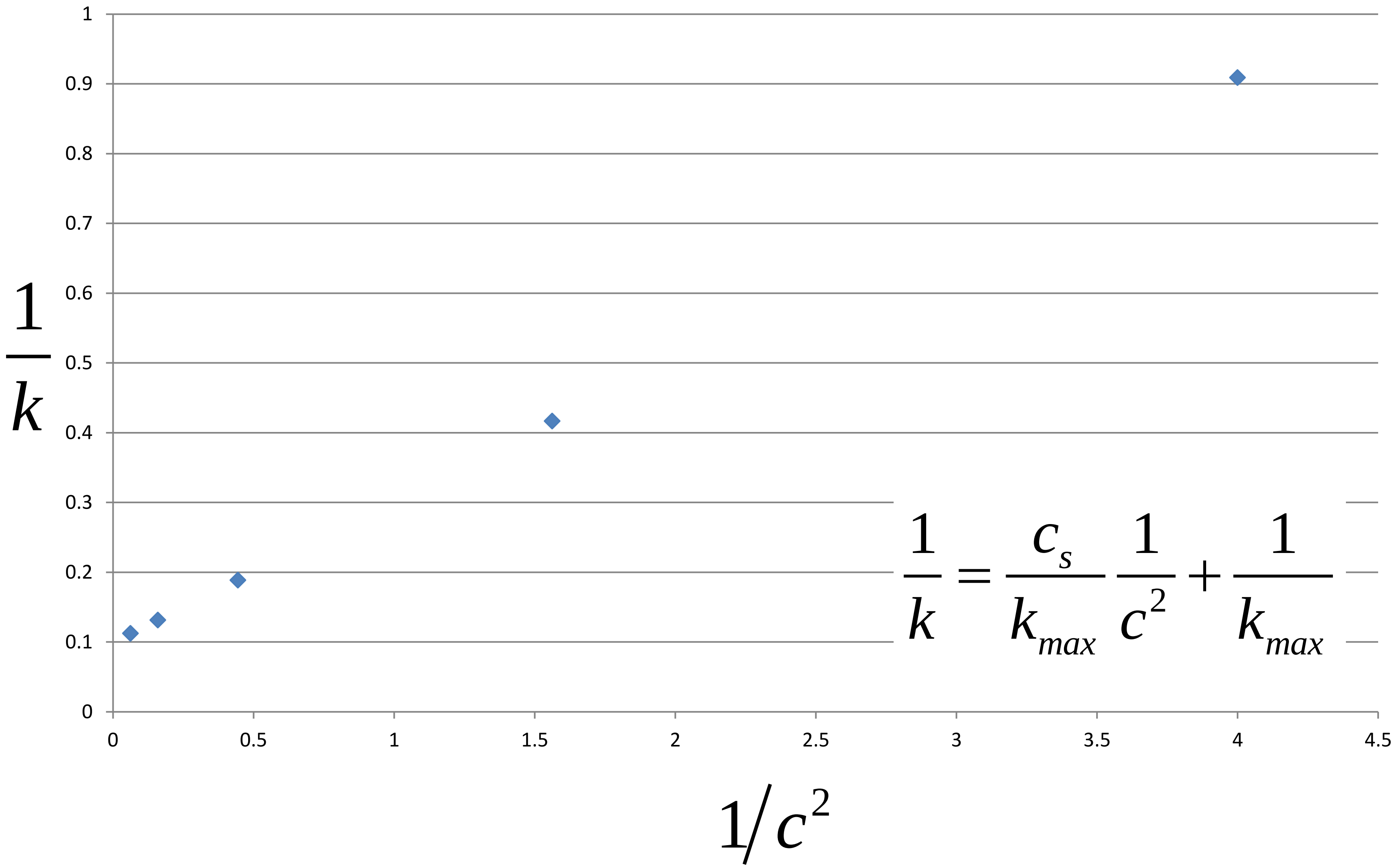
Linealización

$$k = \frac{k_{max} c^2}{c_s + c^2}$$

$$\frac{1}{k} = \frac{c_s + c^2}{k_{max} c^2} \quad \Rightarrow \quad \frac{1}{k} = \frac{c_s}{k_{max} c^2} + \frac{c^2}{k_{max} c^2}$$

$$\frac{1}{k} = \frac{c_s}{k_{max}} \frac{1}{c^2} + \frac{1}{k_{max}}$$

$$\frac{1}{k} = \alpha \frac{1}{c^2} + \beta$$



$$\frac{1}{k} = \frac{c_s}{k_{max}} \frac{1}{c^2} + \frac{1}{k_{max}}$$

$x =$

$A =$

$y =$

c	k
0.5	1.1
0.8	2.4
1.5	5.3
2.5	7.6
4	8.9

$$\frac{1}{k} = \frac{c_s}{k_{max}} \frac{1}{c^2} + \frac{1}{k_{max}}$$

$$\frac{1}{k} = \alpha \frac{1}{c^2} + \beta$$

$$x = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

$$A = \begin{pmatrix} \frac{1}{0.5^2} & 1 \\ \frac{1}{0.8^2} & 1 \\ \frac{1}{1.5^2} & 1 \\ \frac{1}{2.5^2} & 1 \\ \frac{1}{4^2} & 1 \end{pmatrix} \qquad y = \begin{pmatrix} \frac{1}{1.1} \\ \frac{1}{2.4} \\ \frac{1}{5.3} \\ \frac{1}{7.6} \\ \frac{1}{8.9} \end{pmatrix}$$

c	k
0.5	1.1
0.8	2.4
1.5	5.3
2.5	7.6
4	8.9

$$\frac{1}{k} = \frac{c_s}{k_{max}} \frac{1}{c^2} + \frac{1}{k_{max}}$$

$$\frac{1}{k} = \alpha \frac{1}{c^2} + \beta$$

Ecuaciones Normales:

$$x = (A'A) \backslash (A'y)$$

$$A = \begin{pmatrix} \frac{1}{0.5^2} & 1 \\ \frac{1}{0.8^2} & 1 \\ \frac{1}{1.5^2} & 1 \\ \frac{1}{2.5^2} & 1 \\ \frac{1}{4^2} & 1 \end{pmatrix} \qquad y = \begin{pmatrix} \frac{1}{1.1} \\ \frac{1}{2.4} \\ \frac{1}{5.3} \\ \frac{1}{7.6} \\ \frac{1}{8.9} \end{pmatrix}$$

$$A' = \begin{pmatrix} \frac{1}{0.5^2} & \frac{1}{0.8^2} & \frac{1}{1.5^2} & \frac{1}{2.5^2} & \frac{1}{4^2} \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

$$(A'A) = \begin{pmatrix} 18.668443 & 6.229444 \\ 6.229444 & 5 \end{pmatrix} \qquad (A'y) = \begin{pmatrix} 4.399337 \\ 1.758375 \end{pmatrix}$$

Nuevo sistema de ecuaciones

$$\begin{pmatrix} 18.668443 & 6.229444 \\ 6.229444 & 5 \end{pmatrix} \begin{pmatrix} \frac{c_s}{k_{max}} \\ \frac{1}{k_{max}} \end{pmatrix} = \begin{pmatrix} 4.399337 \\ 1.758375 \end{pmatrix}$$

2x2 se puede resolver por cualquier método

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--> (A'*A)\(A'*y)

ans =

  0.202488989680293
  0.099396281429149
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$$\alpha = \frac{c_s}{k_{max}} = 0.20248898$$

$$\beta = \frac{1}{k_{max}} = 0.099396281$$

$$\frac{c_s}{k_{max}} = 0.20248898$$

$$\frac{1}{k_{max}} = 0.099396281$$

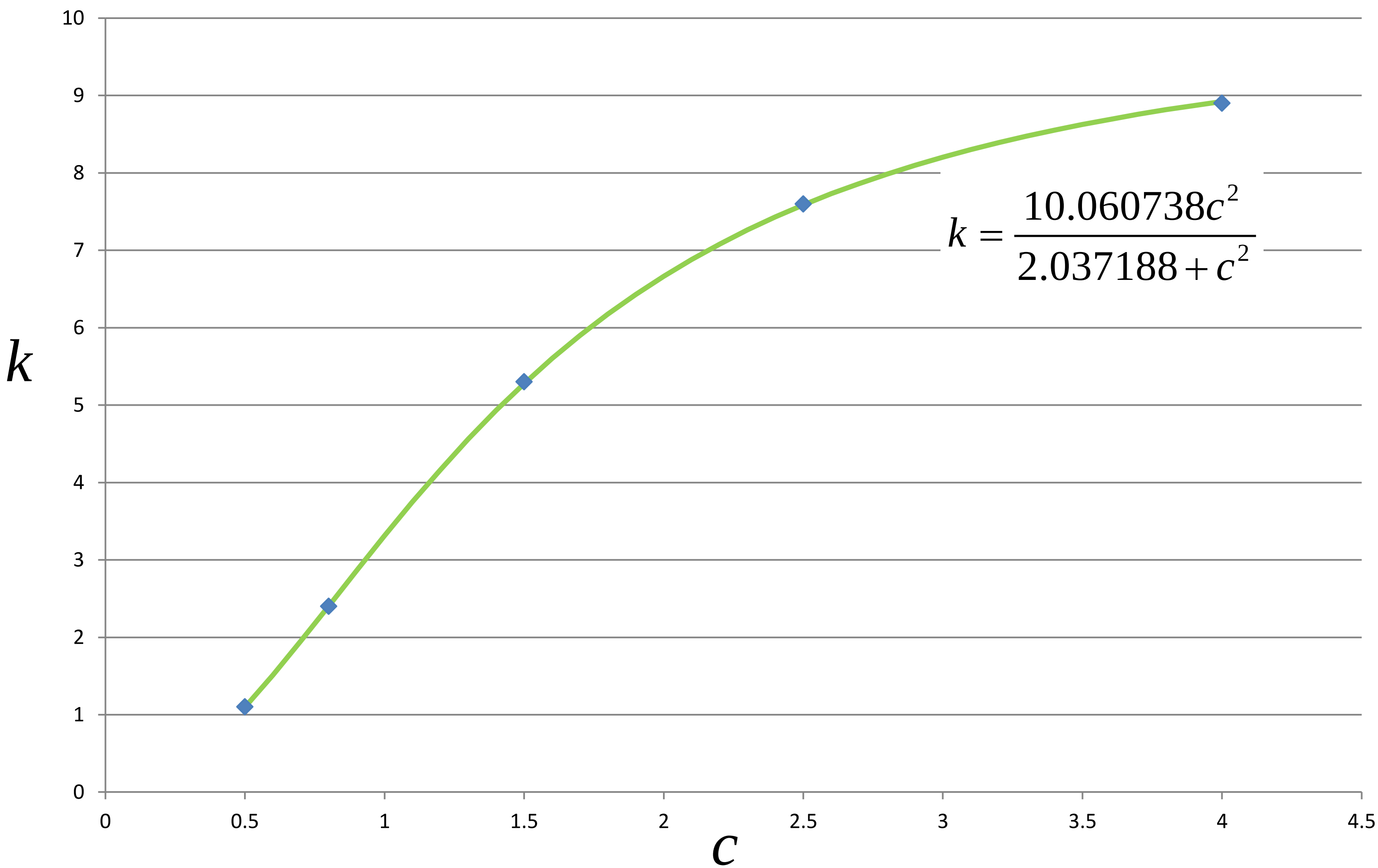
$\Rightarrow$ $k_{max} = 10.060738$ $\Rightarrow$ $c_s = 2.037188$

Finalmente nuestro modelo es:

$$k = \frac{10.060738c^2}{2.037188 + c^2}$$

Ahora podemos predecir el valor de la tasa de crecimiento a c=2mg/l:

$$k\Big|_{2mg/l} = \frac{10.060738 \times 2^2}{2.037188 + 2^2} = 6.665843\,\frac{bact}{dia}$$



10
9
8
7
6
5
4
3
2
1
0
k
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
c
$k = \frac{10.060738c^2}{2.037188 + c^2}$