Unidad 2: Método de Thomas

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Método de Thomas

- El método de Thomas se utiliza para la resolución de sistemas de ecuaciones algebraicas lineales cuya matriz de coeficientes resulta tridiagonal.

$$\begin{bmatrix} B_1 & C_1 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\ A_2 & B_2 & C_2 & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & A_3 & B_3 & C_3 & \cdots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 0 & A_{n-1} & B_{n-1} & C_{n-1} \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 & A_n & B_n \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_{n-1} \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_{n-1} \\ b_n \end{bmatrix}$$

Matriz Tridiagonal

Método de Thomas

$$\begin{bmatrix} B_1 & C_1 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\ A_2 & B_2 & C_2 & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & A_3 & B_3 & C_3 & \cdots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 0 & A_{n-1} & B_{n-1} & C_{n-1} \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 & A_n & B_n \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_{n-1} \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_{n-1} \\ b_n \end{bmatrix}$$

Fila 1:

$$B_1 x_1 + C_1 x_2 = b_1$$

$$B_1 x_1 + C_1 x_2 = b_1$$

$$x_1 = \frac{b_1}{B_1} - \frac{C_1}{B_1} x_2$$

$$Q_1 = \frac{b_1}{B_1} \quad y \quad P_1 = \frac{C_1}{B_1} \Rightarrow x_1 = Q_1 - P_1 x_2$$

Método de Thomas

$$\begin{bmatrix} B_1 & C_1 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\ A_2 & B_2 & C_2 & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & A_3 & B_3 & C_3 & \cdots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 0 & A_{n-1} & B_{n-1} & C_{n-1} \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 & A_n & B_n \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_{n-1} \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_{n-1} \\ b_n \end{bmatrix}$$

Fila 2: $A_2 x_1 + B_2 x_2 + C_2 x_3 = b_2$

Método de Thomas

$$A_2 x_1 + B_2 x_2 + C_2 x_3 = b_2$$

$$x_1 = Q_1 - P_1 x_2$$ de la Fila 1

$$A_2\left(Q_1 - P_1 x_2\right) + B_2 x_2 + C_2 x_3 = b_2$$

$$A_2 Q_1 - A_2 P_1 x_2 + B_2 x_2 + C_2 x_3 = b_2$$

$$A_2 Q_1 + \left(B_2 - A_2 P_1\right) x_2 + C_2 x_3 = b_2 \quad \Rightarrow \quad x_2 = \frac{b_2 - A_2 Q_1}{B_2 - A_2 P_1} - \frac{C_2}{B_2 - A_2 P_1} x_3$$

$$Q_2 = \frac{b_2 - A_2 Q_1}{B_2 - A_2 P_1} \quad y \quad P_2 = \frac{C_2}{B_2 - A_2 P_1} \Rightarrow x_2 = Q_2 - P_2 x_3$$

Método de Thomas

$$\begin{bmatrix} B_1 & C_1 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\ A_2 & B_2 & C_2 & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & A_3 & B_3 & C_3 & \cdots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 0 & A_{n-1} & B_{n-1} & C_{n-1} \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 & A_n & B_n \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_{n-1} \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_{n-1} \\ b_n \end{bmatrix}$$

Fila 3: $A_3 x_2 + B_3 x_3 + C_3 x_4 = b_3$

$$A_3 x_2 + B_3 x_3 + C_3 x_4 = b_3$$

$$x_2 = Q_2 - P_2 x_3 \text{ (de la Fila 2)}$$

$$A_3 \left(Q_2 - P_2 x_3 \right) + B_3 x_3 + C_3 x_4 = b_3$$

$$A_3 Q_2 - A_3 P_2 x_3 + B_3 x_3 + C_3 x_4 = b_3$$

$$A_3 Q_2 + \left(B_3 - A_3 P_2 \right) x_3 + C_3 x_4 = b_3 \quad \Rightarrow \quad x_3 = \frac{b_3 - A_3 Q_2}{B_3 - A_3 P_2} - \frac{C_3}{B_3 - A_3 P_2} x_4$$

$$x_3 = Q_3 - P_3 x_4$$

Método de Thomas

$$Q_2 = \frac{b_2 - A_2 Q_1}{B_2 - A_2 P_1} \Rightarrow Q_3 = \frac{b_3 - A_3 Q_2}{B_3 - A_3 P_2} \Rightarrow Q_i = \frac{b_i - A_i Q_{i-1}}{B_i - A_i P_{i-1}}$$

$$P_2 = \frac{C_2}{B_2 - A_2 P_1} \Rightarrow P_3 = \frac{C_3}{B_3 - A_3 P_2} \Rightarrow P_i = \frac{C_i}{B_i - A_i P_{i-1}}$$

$$x_2 = Q_2 - P_2 x_3 \Rightarrow x_3 = Q_3 - P_3 x_4 \Rightarrow x_i = Q_i - P_i x_{i+1}$$

Hasta la fila n-1

Método de Thomas

$$\begin{bmatrix} B_1 & C_1 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\ A_2 & B_2 & C_2 & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & A_3 & B_3 & C_3 & \cdots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 0 & A_{n-1} & B_{n-1} & C_{n-1} \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 & A_n & B_n \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_{n-1} \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_{n-1} \\ b_n \end{bmatrix}$$

Fila n: $A_n x_{n-1} + B_n x_n = b_n$

Método de Thomas

$$A_n x_{n-1} + B_n x_n = b_n$$

$$x_{n-1} = Q_{n-1} - P_{n-1} x_n$$ (de la fila "n-1")

$$\left.\begin{aligned} A_n\left(Q_{n-1} - P_{n-1}x_n\right) + B_n x_n &= b_n \\ A_n Q_{n-1} - A_n P_{n-1} x_n + B_n x_n &= b_n \\ A_n Q_{n-1} + \left(B_n - A_n P_{n-1}\right) x_n &= b_n \end{aligned}\right\} \quad \boxed{x_n = \frac{b_n - A_n Q_{n-1}}{B_n - A_n P_{n-1}} = Q_n}$$

Método de Thomas

$$P_1 = \frac{C_1}{B_1} \quad y \quad Q_1 = \frac{b_1}{B_1}$$

$$P_i = \frac{C_i}{B_i - A_i P_{i-1}} \quad y \quad Q_i = \frac{b_i - A_i Q_{i-1}}{B_i - A_i P_{i-1}}$$

$$Q_n = \frac{b_n - A_n Q_{n-1}}{B_n - A_n P_{n-1}} \Rightarrow x_n = Q_n$$

$$\begin{bmatrix} B_1 & C_1 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\ A_2 & B_2 & C_2 & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & A_3 & B_3 & C_3 & \cdots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 0 & A_{n-1} & B_{n-1} & C_{n-1} \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 & A_n & B_n \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_{n-1} \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_{n-1} \\ b_n \end{bmatrix}$$

Método de Thomas

$$x_i = Q_i - P_i x_{i+1}$$ → Para cualquier fila

$$x_{n-1} = Q_{n-1} - P_{n-1} x_{n-1+1}$$ → Fila (n-1)

$$x_{n-1} = Q_{n-1} - P_{n-1} x_n$$

¿Que conozco?

Método de Thomas

Ej: Aplicación en un sistema 4x4

$$\begin{bmatrix} B_1 & C_1 & 0 & 0 \\ A_2 & B_2 & C_2 & 0 \\ 0 & A_3 & B_3 & C_3 \\ 0 & 0 & A_4 & B_4 \end{bmatrix} \times \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}$$

$$P_1 = \frac{C_1}{B_1} \qquad Q_1 = \frac{b_1}{B_1}$$

$$\Rightarrow$$

$$P_2 = \frac{C_2}{B_2 - A_2 P_1} \qquad Q_2 = \frac{b_2 - A_2 Q_1}{B_2 - A_2 P_1}$$

$$\Rightarrow$$

$$P_3 = \frac{C_3}{B_3 - A_3 P_2} \qquad Q_3 = \frac{b_3 - A_3 Q_2}{B_3 - A_3 P_2}$$

$$\Rightarrow$$

$$Q_4 = \frac{b_4 - A_4 Q_3}{B_4 - A_4 P_3}$$

$$\Downarrow$$

$$x_1 = Q_1 - P_1 x_2 \Leftarrow x_2 = Q_2 - P_2 x_3 \Leftarrow x_3 = Q_3 - P_3 x_4 \Leftarrow x_4 = Q_4$$

Método de Thomas

$$P_1 = \frac{C_1}{B_1} \qquad Q_1 = \frac{b_1}{B_1}$$

$$P_2 = \frac{C_2}{B_2 - A_2 P_1} \qquad Q_2 = \frac{b_2 - A_2 Q_1}{B_2 - A_2 P_1}$$

$$P_3 = \frac{C_3}{B_3 - A_3 P_2} \qquad Q_3 = \frac{b_3 - A_3 Q_2}{B_3 - A_3 P_2}$$

$$Q_4 = \frac{b_4 - A_4 Q_3}{B_4 - A_4 P_3}$$

$$x_1 = Q_1 - P_1 x_2 \leftarrow x_2 = Q_2 - P_2 x_3 \leftarrow x_3 = Q_3 - P_3 x_4 \leftarrow x_4 = Q_4$$

$$i = 2:(n-1)$$

$$i = (n-1):-1:1$$

$$P_1 = \frac{C_1}{B_1} \qquad Q_1 = \frac{b_1}{B_1}$$

$$P_i = \frac{C_i}{B_i - A_i P_{i-1}} \qquad Q_i = \frac{b_i - A_i Q_{i-1}}{B_i - A_i P_{i-1}}$$

$$Q_n = \frac{b_n - A_n Q_{n-1}}{B_n - A_n P_{n-1}} \qquad x_n = Q_n$$

$$x_i = Q_i - P_i x_{i+1}$$

Método de Thomas (scilab)

```
function x=Thomas(M,b)
A=diag(M,-1);
B=diag(M,0);
C=diag(M,1);
//conviene agregar ceros en A y C
A=[0;A];
C=[C;0];
n=length(b);




endfunction
```

$$P_1 = \frac{C_1}{B_1} \qquad Q_1 = \frac{b_1}{B_1}$$

$$P_i = \frac{C_i}{B_i - A_i P_{i-1}} \qquad Q_i = \frac{b_i - A_i Q_{i-1}}{B_i - A_i P_{i-1}} \qquad i = 2:(n-1)$$

$$Q_n = \frac{b_n - A_n Q_{n-1}}{B_n - A_n P_{n-1}}$$

$$x_n = Q_n$$

$$x_i = Q_i - P_i x_{i+1} \qquad i = (n-1):-1:1$$