# Unidad 2: Sustitución hacia atrás y hacia delante

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# Sustitución hacia atrás

A =

| | | | |
|---|---|---|---|
| 16.0000 | 12.0000 | 19.0000 | 19.0000 |
| 0 | -12.5000 | -2.3750 | -12.3750 |
| 0 | 0 | -0.0400 | 10.1600 |
| 0 | 0 | 0 | -450.2500 |

b =

| |
|---|
| 4.0000 |
| 4.5000 |
| 7.7600 |
| -328.5000 |

| | | | | |
|---|---|---|---|---|
| | | | | $x_1$<br>$x_2$<br>$x_3$<br>$x_4$ |
| 16.0000 | 12.0000 | 19.0000 | 19.0000 | 4.0000 |
| 0 | -12.5000 | -2.3750 | -12.3750 | 4.5000 |
| 0 | 0 | -0.0400 | 10.1600 | 7.7600 |
| 0 | 0 | 0 | -450.2500 | -328.5000 |

# Sustitución hacia atrás

**¡Matriz de coeficientes triangular superior!**

$x_1$
$x_2$
$x_3$
$x_4$

| | | | | |
|---|---|---|---|---|
| 16.0000 | 12.0000 | 19.0000 | 19.0000 | 4.0000 |
| 0 | -12.5000 | -2.3750 | -12.3750 | 4.5000 |
| 0 | 0 | -0.0400 | 10.1600 | 7.7600 |
| 0 | 0 | 0 | -450.2500 | -328.5000 |

$$A_{1,1}x_1 + A_{1,2}x_2 + A_{1,3}x_3 + A_{1,4}x_4 = b_1$$

$$A_{2,2}x_2 + A_{2,3}x_3 + A_{2,4}x_4 = b_2$$

$$A_{3,3}x_3 + A_{3,4}x_4 = b_3$$

$$A_{4,4}x_4 = b_4$$

# Sustitución hacia atrás

$$A_{1,1}x_1 + A_{1,2}x_2 + A_{1,3}x_3 + A_{1,4}x_4 = b_1$$

$$A_{2,2}x_2 + A_{2,3}x_3 + A_{2,4}x_4 = b_2$$

$$A_{3,3}x_3 + A_{3,4}x_4 = b_3$$

$$A_{4,4}x_4 = b_4$$

$$x_4 = \frac{b_4}{A_{4,4}}$$

$$x_3 = \frac{b_3 - A_{3,4}x_4}{A_{3,3}}$$

$$x_2 = \frac{b_2 - (A_{2,3}x_3 + A_{2,4}x_4)}{A_{2,2}}$$

$$x_1 = \frac{b_1 - (A_{1,2}x_2 + A_{1,3}x_3 + A_{1,4}x_4)}{A_{1,1}}$$

# Sustitución hacia atrás

$$x_4 = \frac{b_4}{A_{4,4}}$$

$$x_n = \frac{b_n}{A_{n,n}}$$

# Sustitución hacia atrás

$$x_3 = \frac{b_3 - A_{3,4}x_4}{A_{3,3}}$$

$$x_2 = \frac{b_2 - \left(A_{2,3}x_3 + A_{2,4}x_4\right)}{A_{2,2}}$$

$$x_1 = \frac{b_1 - \left(A_{1,2}x_2 + A_{1,3}x_3 + A_{1,4}x_4\right)}{A_{1,1}}$$

# Sustitución hacia atrás

$$x_3 = \frac{b_3 - A_{3,4}x_4}{A_{3,3}}$$

| | |
|---|---|
| | $x_4$ |
| $A_{3,4}$ | $A_{3,4}x_4$ |

# Sustitución hacia atrás



$$x_2 = \frac{b_2 - \left(A_{2,3}x_3 + A_{2,4}x_4\right)}{A_{2,2}}$$

| | | |
|---|---|---|
| | | $x_3$ |
| | | $x_4$ |
| $A_{2,3}$ | $A_{2,4}$ | $A_{2,3}x_3 + A_{2,4}x_4$ |

# Sustitución hacia atrás



$$x_1 = \frac{b_1 - \left( A_{1,2}x_2 + A_{1,3}x_3 + A_{1,4}x_4 \right)}{A_{1,1}}$$

| | $x_2$<br>$x_3$<br>$x_4$ |
|---|---|
| $A_{1,2}$ $A_{1,3}$ $A_{1,4}$ | $A_{1,2}x_2 + A_{1,3}x_3 + A_{1,4}x_4$ |

# Sustitución hacia atrás



$$x_1 = \frac{b_1 - \left(A_{1,2}x_2 + A_{1,3}x_3 + A_{1,4}x_4\right)}{A_{1,1}}$$

$$x_i = \frac{b_i - A_{i,i+1:n} \times x_{i+1:n}}{A_{i,i}}$$

## ¿Cómo se genera cada vector en Scilab?

$$A_{i,i+1:n} \times x_{i+1:n}$$

$$\begin{array}{c|c} & \begin{matrix} x_{i+1} \\ \vdots \\ x_{n-1} \\ x_n \end{matrix} \\ \hline \begin{matrix} A_{i,i+1} & \cdots & A_{i,n-1} & A_{i,n} \end{matrix} & A_{i,i+1}x_{i+1} + \cdots + A_{i,n-1}x_{n-1} + A_{i,n}x_n \end{array}$$

# Sustitución hacia atrás

$$\left[ A_{i,i+1} \;\cdots\; A_{i,n-1} \;\; A_{i,n} \right]$$

¿En Scilab?

A(i,[i+1:n])

¿En Scilab?

$$\begin{bmatrix} x_{i+1} \\ \vdots \\ x_{n-1} \\ x_n \end{bmatrix}$$

x([i+1:n],1)

# Sustitución hacia atrás

$$A_{i,i+1}x_{i+1} + \cdots + A_{i,n-1}x_{n-1} + A_{i,n}x_n$$

¿En Scilab?

```
A(i,[i+1:n])*x([i+1:n],1)
```

# Sustitución hacia atrás



$$x_i = \frac{b_i - A_{i,i+1:n} \times x_{i+1:n}}{A_{i,i}}$$

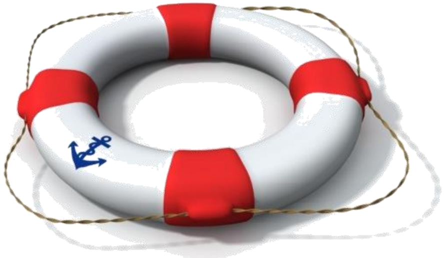
¿En Scilab?

```
x(i,1)=(b(i) - A(i,[i+1:n])*x([i+1:n],1))/A(i,i);
```

```
x(i,1)=(b(i) - A(i,[i+1:n])*x([i+1:n],1))/A(i,i);
```

¿Cómo varia *i* ?

$$x_4 = \frac{b_4}{A_{4,4}} \rightarrow x_3 = \frac{b_3 - A_{3,4}x_4}{A_{3,3}} \rightarrow x_2 = \frac{b_2 - (A_{2,3}x_3 + A_{2,4}x_4)}{A_{2,2}}$$



$$\downarrow$$

$$x_1 = \frac{b_1 - (A_{1,2}x_2 + A_{1,3}x_3 + A_{1,4}x_4)}{A_{1,1}}$$

# Sustitución hacia atrás



```
function x=SA(A, b)
[n c]=size(A);
x(n,1)=b(n)/A(n,n);
for i=n-1:-1:1
 x(i,1)=(b(i) - A(i,[i+1:n])*x([i+1:n],1))/A(i,i);
end
endfunction
```

# Sustitución hacia atrás



```
A =[16   12   19   19
    18    1   19    9
     2    5    3   16
    18   10   19    2];
b =[4   9   7   9]';

[A b]=gaussiana(A,b)

x=SA(A,b)
```

```
A  =

   16.    12.      19.       19.
   0.    -12.5   -2.375   -12.375
   0.     0.     -0.04     10.16
   0.     0.      0.      -450.25
b  =

   4.
   4.5
   7.76
 -328.50000
```

```
x  =

   9.2690172
   0.5674625
 -8.6829539
   0.7295947
```

# Sustitución hacia atrás



```
A =[16   12   19   19
    18    1   19    9
     2    5    3   16
    18   10   19    2];
b =[4   9   7   9]';

[A,b]=gaussianaPP(A,b)

x=SA(A,b)
```

```
A  =

   18.    1.           19.          9.
   0.     11.111111    2.1111111    11.
   0.     0.          -1.71        -15.91
   0.     0.           0.           10.532164
b  =

   9.
  -4.
   3.24
   7.6842105

 x  =

   9.2690172
   0.5674625
  -8.6829539
   0.7295947
```

# Sustitución hacia atrás

```
A =[16    12    19    19
    18     1    19     9
     2     5     3    16
    18    10    19     2];
b =[4    9    7    9]';

[A,b,P]=gaussianaPT(A,b)

xn=SA(A,b)

x= P*xn
```

```
 x  =

   9.2690172
   0.5674625
  -8.6829539
   0.7295947
```

```
A  =

   19.    19.    12.           16.
   0.    -17.   -2.            2.
   0.     0.    -9.8235294     0.8235294
   0.     0.     0.            1.1352033
b  =

   4.
   5.
   2.0588235
   10.522219
P  =

   0.    0.    0.    1.
   0.    0.    1.    0.
   1.    0.    0.    0.
   0.    1.    0.    0.
xn  =

  -8.6829539
   0.7295947
   0.5674625
   9.2690172
```

# Sustitución hacia delante



**¡Matriz de coeficientes triangular inferior!**

$$A_{1,1}x_1 = b_1$$

$$A_{2,1}x_1 + A_{2,2}x_2 = b_2$$

$$A_{3,1}x_1 + A_{3,2}x_2 + A_{3,3}x_3 = b_3$$

$$A_{4,1}x_1 + A_{4,2}x_2 + A_{4,3}x_3 + A_{4,4}x_4 = b_4$$

$$\begin{pmatrix} A_{1,1} & 0 & 0 & 0 \\ A_{2,1} & A_{2,2} & 0 & 0 \\ A_{3,1} & A_{3,2} & A_{3,3} & 0 \\ A_{4,1} & A_{4,2} & A_{4,3} & A_{4,4} \end{pmatrix}$$

$$x_1 = \frac{b_1}{A_{1,1}} \qquad x_2 = \frac{b_2 - A_{2,1}x_1}{A_{2,2}} \qquad x_3 = \frac{b_3 - \left(A_{3,1}x_1 + A_{3,2}x_2\right)}{A_{3,3}}$$

$$x_4 = \frac{b_4 - \left(A_{4,1}x_1 + A_{4,2}x_2 + A_{4,3}x_3\right)}{A_{4,4}} \quad \Rightarrow x_i = \frac{b_i - A_{i,1:i-1} \times x_{1:i-1}}{A_{i,i}}$$

# Sustitución hacia delante

```
function x=SD(A, b)
[n c]=size(A);
x(1,1)=b(1)/A(1,1);
for i=2:n
 x(i,1)=(b(i) - A(i,[1:i-1])*x([1:i-1],1))/A(i,i);
end
endfunction
```