Aproximación de derivadas con mayor exactitud y mayor orden para valores discretos

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Conclusión de la presentación anterior

$$f'(x_i) \cong \frac{f(x_{i+1}) - f(x_i)}{(x_{i+1} - x_i)}$$

$$f'(x_i) \cong \frac{f(x_i + h) - f(x_i)}{h} \Rightarrow f'(x_i) \cong \frac{f(x_{i+1}) - f(x_i)}{h}$$

Si los datos se encuentran equiespaciados podemos utilizar las expresiones que analizamos anteriormente

Aproximación central

$$f(x_{i+1}) = f(x_i) + f'(x_i)(x_{i+1}-x_i) + \frac{f''(x_i)}{2!}(x_{i+1}-x_i)^2 + \frac{f'''(x_i)}{3!}(x_{i+1}-x_i)^3 + \frac{f''''(x_i)}{4!}(x_{i+1}-x_i)^4 + \frac{f'''''(x_i)}{5!}(x_{i+1}-x_i)^5 + \ldots$$

$$f(x_{i-1}) = f(x_i) + f'(x_i)(x_{i-1}-x_i) + \frac{f''(x_i)}{2!}(x_{i-1}-x_i)^2 + \frac{f'''(x_i)}{3!}(x_{i-1}-x_i)^3 + \frac{f''''(x_i)}{4!}(x_{i-1}-x_i)^4 + \frac{f'''''(x_i)}{5!}(x_{i-1}-x_i)^5 + \ldots$$

$$f(x_{i+1}) - f(x_{i-1}) = f'(x_i)\Big((x_{i+1}-x_i) - (x_{i-1}-x_i)\Big) + \frac{f''(x_i)}{2!}\Big((x_{i+1}-x_i)^2 - (x_{i-1}-x_i)^2\Big) + \frac{f'''(x_i)}{3!}\Big((x_{i+1}-x_i)^3 - (x_{i-1}-x_i)^3\Big) + \ldots$$

$$f(x_{i+1}) - f(x_{i-1}) = f'(x_i)\big(x_{i+1}-x_{i-1}\big) + \frac{f''(x_i)}{2!}\Big((x_{i+1}-x_i)^2 - (x_{i-1}-x_i)^2\Big) + \frac{f'''(x_i)}{3!}\Big((x_{i+1}-x_i)^3 - (x_{i-1}-x_i)^3\Big) + \ldots$$

$$f(x_{i+1}) - f(x_{i-1}) = f'(x_i)h + \frac{f''(x_i)}{2!}\Big((h)^2 - (-h)^2\Big) + \frac{f'''(x_i)}{3!}\Big((h)^3 - (-h)^3\Big) + \ldots$$

$$f(x_{i+1}) - f(x_{i-1}) = f'(x_i)h + \cancel{\frac{f''(x_i)}{2!}0} + \frac{f'''(x_i)}{3!}2h^3 + \ldots$$

¡Se reduce el error!

$$f'(x_i) \cong \frac{f(x_{i+1}) - f(x_{i-1})}{h}$$

Mayor exactitud

$$f'(x_0) = \frac{48f(x_0+h) - 36f(x_0+2h) + 16f(x_0+3h) - 3f(x_0+4h) - 25f(x_0)}{12h}$$

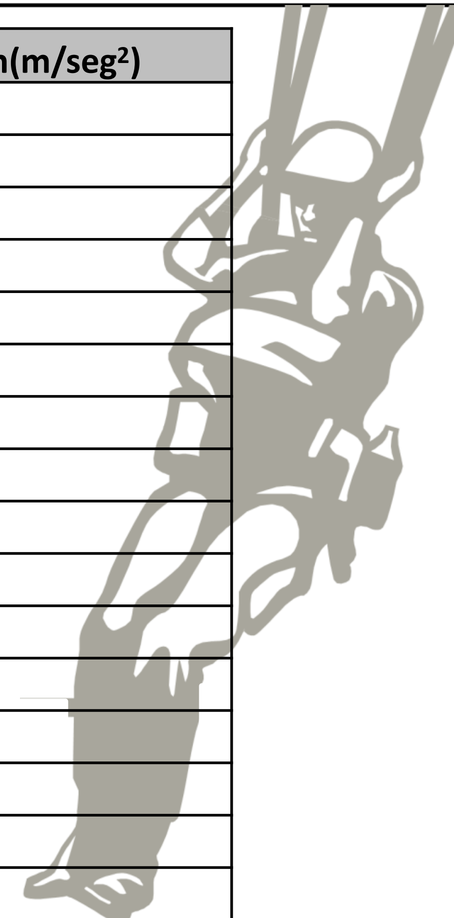
$$f'(x_i) = \frac{48f(x_{i+1}) - 36f(x_{i+2}) + 16f(x_{i+3}) - 3f(x_{i+4}) - 25f(x_i)}{12h}$$

$$f'(x_0) = \frac{-3f(x_0-h) + 18f(x_0+h) - 6f(x_0+2h) + f(x_0+3h) - 10f(x_0)}{12h}$$

$$f'(x_i) = \frac{-3f(x_{i-1}) + 18f(x_{i+1}) - 6f(x_{i+2}) + f(x_{i+3}) - 10f(x_i)}{12h}$$

Aplicación

Tiempo (seg)	Distancia (m)	Velocidad (m/seg)	Aceleracion(m/seg²)
0	0		
2	17.41		
4	62.27		
6	126.17		
8	203.24		
10	289.44		
12	381.96		
14	478.86		
16	578.80		
18	680.84		
20	784.34		
22	888.85		
24	994.05		
26	1099.74		
28	1205.76		
30	1312.02		
32	1418.44		
34	1524.97		
36	1631.58		
38	1738.24		
40	1844.93		



Aplicación

Tiempo (seg)	Distancia (m)	Velocidad (m/seg)	Aceleracion(m/seg²)
0	0		
2	17.41		
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6			
8			
10			
12			
14			
16			
18			
20			
22			
24			
26			
28			
30			
32			
34	1524.97		
36	1631.58		
38	1738.24		
40	1844.93		

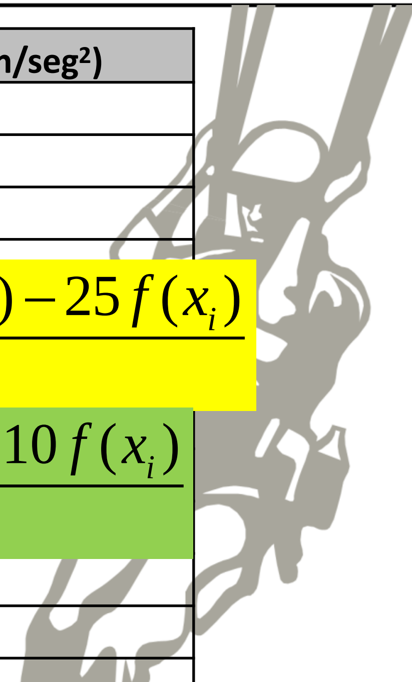
$$f'(x_i)=\frac{48f(x_{i+1})-36f(x_{i+2})+16f(x_{i+3})-3f(x_{i+4})-25f(x_i)}{12h}$$

$$f'(x_i)=\frac{-3f(x_{i-1})+18f(x_{i+1})-6f(x_{i+2})+f(x_{i+3})-10f(x_i)}{12h}$$

$$f'(x_i)=\frac{-8f(x_{i-1})+f(x_{i-2})+8f(x_{i+1})-f(x_{i+2})}{12h}$$

$$f'(x_i)=\frac{3f(x_{i+1})-18f(x_{i-1})+6f(x_{i-2})-f(x_{i-3})+10f(x_i)}{12h}$$

$$f'(x_i)=\frac{-48f(x_{i-1})+36f(x_{i-2})-16f(x_{i-3})+3f(x_{i-4})+25f(x_i)}{12h}$$



Aplicación

Tiempo (seg)	Distancia (m)	Velocidad (m/seg)	Aceleracion(m/seg²)
0	0		
2	17.41		
4	[illegible]		
6			
8			
10			
12			
14			
16			
18			
20			
22			
24			
26			
28			
30			
32			
34	1524.97		
36	1631.58		
38	1738.24		
40	1844.93		

$$f''(x_i) = \frac{-104f(x_{i+1}) + 114f(x_{i+2}) - 56f(x_{i+3}) + 11f(x_{i+4}) + 35f(x_i)}{12h^2}$$

$$f''(x_i) = \frac{11f(x_{i-1}) + 6f(x_{i+1}) + 4f(x_{i+2}) - f(x_{i+3}) - 20f(x_i)}{12h^2}$$

$$f''(x_i) = \frac{16f(x_{i-1}) - f(x_{i-2}) + 16f(x_{i+1}) - f(x_{i+2}) - 30f(x_i)}{12h^2}$$

$$f''(x_i) = \frac{-11f(x_{i+1}) - 6f(x_{i-1}) - 4f(x_{i-2}) + f(x_{i-3}) + 20f(x_i)}{12h^2}$$

$$f''(x_i) = \frac{104f(x_{i-1}) - 114f(x_{i-2}) + 56f(x_{i-3}) - 11f(x_{i-4}) - 35f(x_i)}{12h^2}$$

Implementación en scilab

Matemática Superior Aplicada

```
function fp=data_deriv(f, x , h)
```

fp(1)= $\dfrac{48f(x_{i+1})-36f(x_{i+2})+16f(x_{i+3})-3f(x_{i+4})-25f(x_i)}{12h}$

fp(2)= $\dfrac{-3f(x_{i-1})+18f(x_{i+1})-6f(x_{i+2})+f(x_{i+3})-10f(x_i)}{12h}$

```
for i=3:length(x)- 2
```

fp(i)= $\dfrac{-8f(x_{i-1})+f(x_{i-2})+8f(x_{i+1})-f(x_{i+2})}{12h}$

¡Completar!

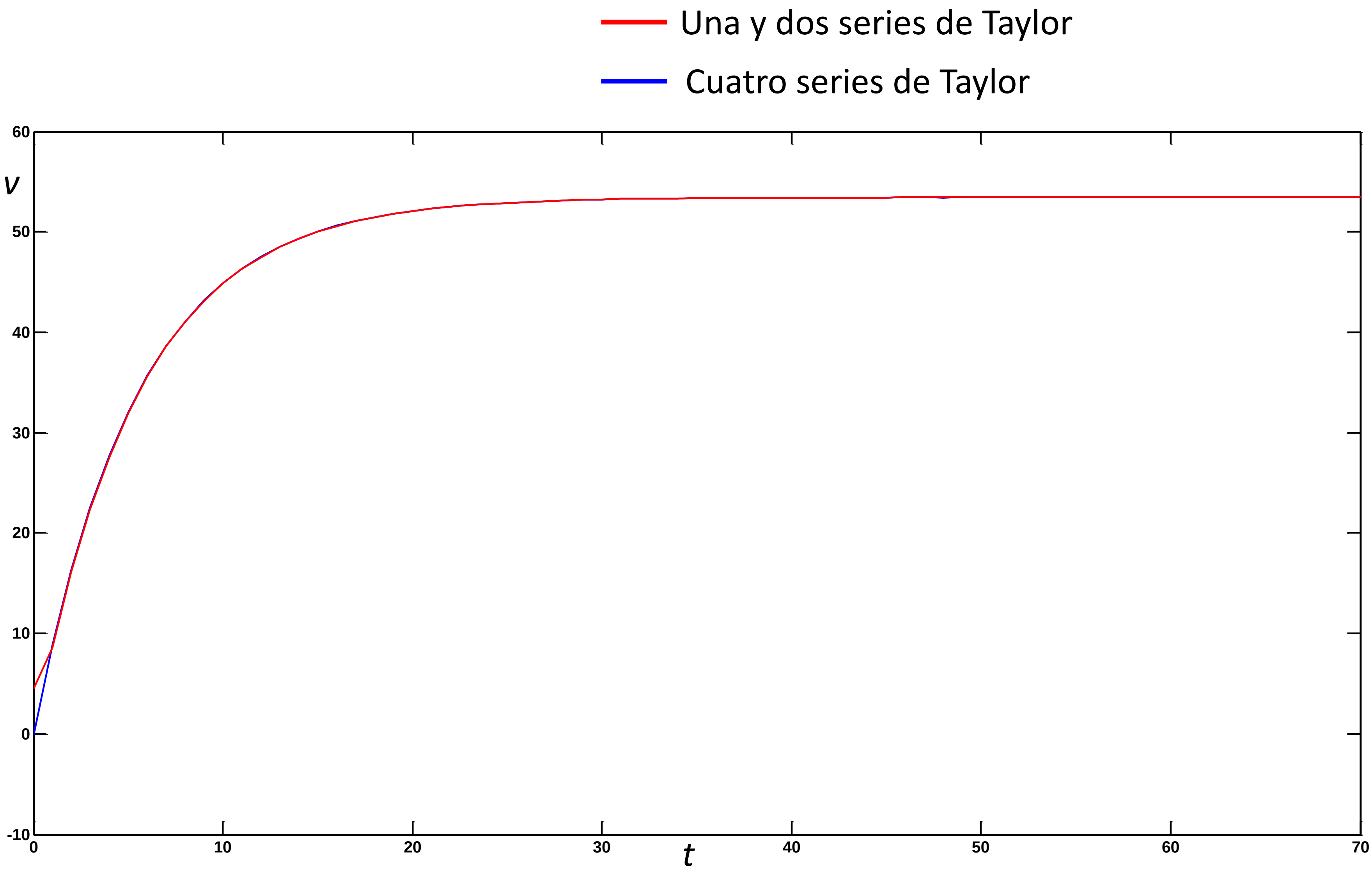
```
end
```

fp(length(x)- 1)= $\dfrac{3f(x_{i+1})-18f(x_{i-1})+6f(x_{i-2})-f(x_{i-3})+10f(x_i)}{12h}$

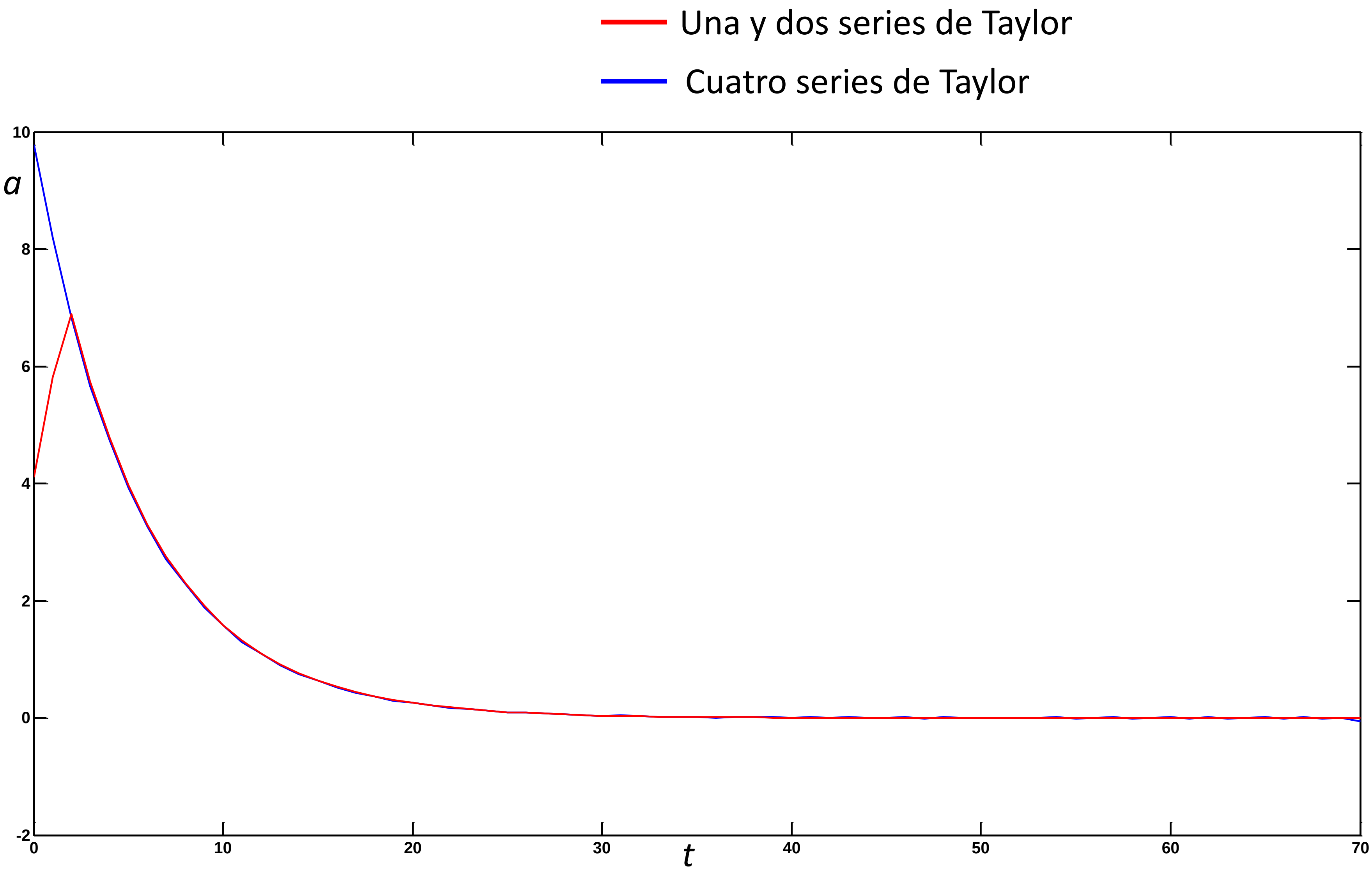
fp(length(x))= $\dfrac{-48f(x_{i-1})+36f(x_{i-2})-16f(x_{i-3})+3f(x_{i-4})+25f(x_i)}{12h}$

```
endfunction
```

Aplicación


Una y dos series de Taylor
Cuatro series de Taylor
v
t
60
50
40
30
20
10
0
-10
0
10
20
30
40
50
60
70

Aplicación


Una y dos series de Taylor
Cuatro series de Taylor
a
t
10
8
6
4
2
0
-2
0
10
20
30
40
50
60
70