# Aproximación de derivadas con valores discretos

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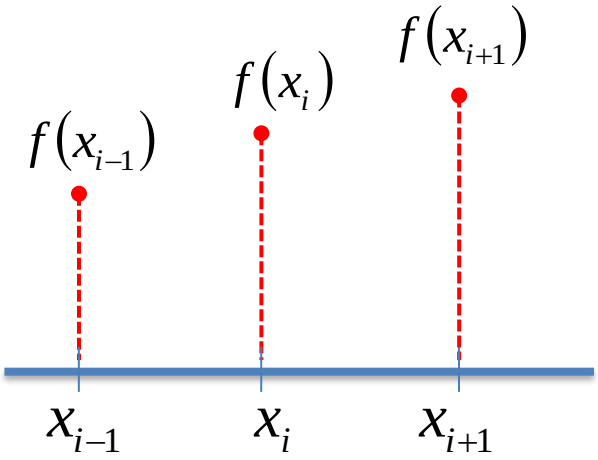
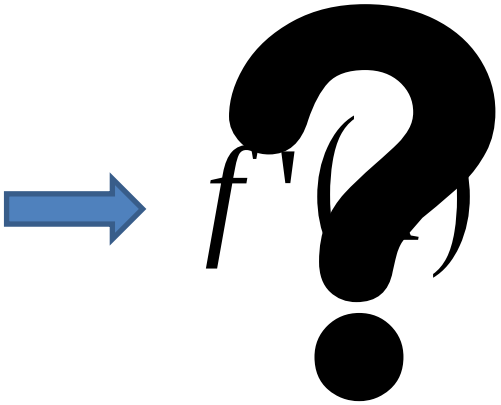
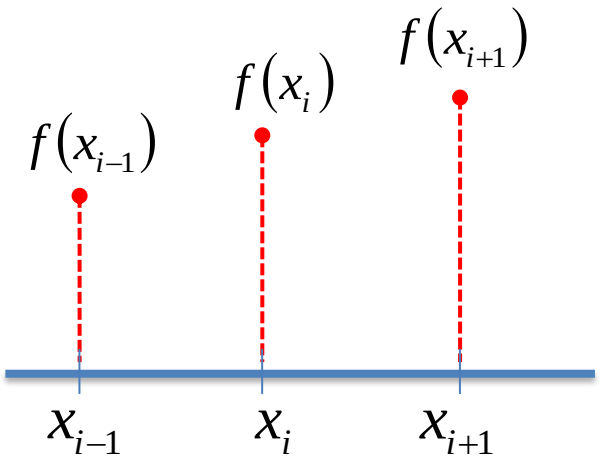
# ¿A que llamamos valores discretos?




| $x$ | $f(x)$ |
|---|---|
| $x_0$ | $f(x_0)$ |
| $x_1$ | $f(x_1)$ |
| $x_2$ | $f(x_2)$ |
| $x_3$ | $f(x_3)$ |
| $\vdots$ | $\vdots$ |
| $x_{i-1}$ | $f(x_{i-1})$ |
| $x_i$ | $f(x_i)$ |
| $x_{i+1}$ | $f(x_{i+1})$ |
| $\vdots$ | $\vdots$ |
| $x_n$ | $f(x_n)$ |



# Aproximación con intervalos discretos




## Serie de Taylor en torno de $x_i$

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(x_i)}{n!}(x - x_i)^n$$

## Podemos conocer el valor en el punto siguiente

$$f(x_{i+1}) = \sum_{n=0}^{\infty} \frac{f^{(n)}(x_i)}{n!}(x_{i+1} - x_i)^n$$

# Aproximación hacia delante

$$f(x_{i+1}) = \sum_{n=0}^{\infty} \frac{f^{(n)}(x_i)}{n!}(x_{i+1} - x_i)^n$$

$$f(x_{i+1}) = f(x_i) + f'(x_i)(x_{i+1} - x_i) + \frac{f''(x_i)}{2!}(x_{i+1} - x_i)^2 + \frac{f'''(x_i)}{3!}(x_{i+1} - x_i)^3 + \frac{f''''(x_i)}{4!}(x_{i+1} - x_i)^4 + \frac{f'''''(x_i)}{5!}(x_{i+1} - x_i)^5 + \ldots$$

$$f'(x_i) = \frac{f(x_{i+1}) - f(x_i)}{(x_{i+1} - x_i)} - \left( \frac{f''(x_i)}{2!}(x_{i+1} - x_i) + \frac{f'''(x_i)}{3!}(x_{i+1} - x_i)^2 + \frac{f''''(x_i)}{4!}(x_{i+1} - x_i)^3 + \frac{f'''''(x_i)}{5!}(x_{i+1} - x_i)^4 + \ldots \right)$$

$$f'(x_i) \cong \frac{f(x_{i+1}) - f(x_i)}{(x_{i+1} - x_i)}$$

# Aproximación hacia atrás

$$f(x_{i-1}) = \sum_{n=0}^{\infty} \frac{f^{(n)}(x_i)}{n!}(x_{i-1} - x_i)^n$$

$$f(x_{i-1}) = f(x_i) + f'(x_i)(x_{i-1} - x_i) + \frac{f''(x_i)}{2!}(x_{i-1} - x_i)^2 + \frac{f'''(x_i)}{3!}(x_{i-1} - x_i)^3 + \frac{f''''(x_i)}{4!}(x_{i-1} - x_i)^4 + \frac{f'''''(x_i)}{5!}(x_{i-1} - x_i)^5 + \ldots$$

$$f'(x_i) = \frac{f(x_{i-1}) - f(x_i)}{(x_{i-1} - x_i)} - \left( \frac{f''(x_i)}{2!}(x_{i-1} - x_i) + \frac{f'''(x_i)}{3!}(x_{i-1} - x_i)^2 + \frac{f''''(x_i)}{4!}(x_{i-1} - x_i)^3 + \frac{f'''''(x_i)}{5!}(x_{i-1} - x_i)^4 + \ldots \right)$$

$$f'(x_i) \cong \frac{f(x_{i-1}) - f(x_i)}{(x_{i-1} - x_i)}$$

# Aproximación central

$$f(x_{i+1}) = \sum_{n=0}^{\infty} \frac{f^{(n)}(x_i)}{n!}(x_{i+1} - x_i)^n \qquad f(x_{i-1}) = \sum_{n=0}^{\infty} \frac{f^{(n)}(x_i)}{n!}(x_{i-1} - x_i)^n$$

$$f(x_{i+1}) = f(x_i) + f'(x_i)(x_{i+1} - x_i) + \frac{f''(x_i)}{2!}(x_{i+1} - x_i)^2 + \frac{f'''(x_i)}{3!}(x_{i+1} - x_i)^3 + \frac{f''''(x_i)}{4!}(x_{i+1} - x_i)^4 + \frac{f'''''(x_i)}{5!}(x_{i+1} - x_i)^5 + \ldots$$

$$f(x_{i-1}) = f(x_i) + f'(x_i)(x_{i-1} - x_i) + \frac{f''(x_i)}{2!}(x_{i-1} - x_i)^2 + \frac{f'''(x_i)}{3!}(x_{i-1} - x_i)^3 + \frac{f''''(x_i)}{4!}(x_{i-1} - x_i)^4 + \frac{f'''''(x_i)}{5!}(x_{i-1} - x_i)^5 + \ldots$$

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$$f(x_{i+1}) - f(x_{i-1}) = f'(x_i)\left((x_{i+1} - x_i) - (x_{i-1} - x_i)\right) + \frac{f''(x_i)}{2!}\left((x_{i+1} - x_i)^2 - (x_{i-1} - x_i)^2\right) + \frac{f'''(x_i)}{3!}\left((x_{i+1} - x_i)^3 - (x_{i-1} - x_i)^3\right) + \ldots$$

$$f(x_{i+1}) - f(x_{i-1}) = f'(x_i)(x_{i+1} - x_{i-1}) + \frac{f''(x_i)}{2!}\left((x_{i+1} - x_i)^2 - (x_{i-1} - x_i)^2\right) + \frac{f'''(x_i)}{3!}\left((x_{i+1} - x_i)^3 - (x_{i-1} - x_i)^3\right) + \ldots$$

$$f'(x_i) \cong \frac{f(x_{i+1}) - f(x_{i-1})}{(x_{i+1} - x_{i-1})}$$

# Aplicación

| $x$ | $f(x)$ |
|---|---|
| $x_0$ | $f(x_0)$ |
| $x_1$ | $f(x_1)$ |
| $x_2$ | $f(x_2)$ |
| $x_3$ | $f(x_3)$ |
| $\vdots$ | $\vdots$ |
| $x_{i-1}$ | $f(x_{i-1})$ |
| $x_i$ | $f(x_i)$ |
| $x_{i+1}$ | $f(x_{i+1})$ |
| $\vdots$ | $\vdots$ |
| $x_n$ | $f(x_n)$ |

$$f'(x_i) \cong \frac{f(x_{i+1}) - f(x_i)}{(x_{i+1} - x_i)}$$

$$f'(x_i) \cong \frac{f(x_{i+1}) - f(x_{i-1})}{(x_{i+1} - x_{i-1})}$$

$$f'(x_i) \cong \frac{f(x_{i-1}) - f(x_i)}{(x_{i-1} - x_i)}$$

# Aplicación



$$f'(x_i) \cong \frac{f(x_{i+1}) - f(x_i)}{(x_{i+1} - x_i)}$$

$$f'(x_i) \cong \frac{f(x_i + h) - f(x_i)}{h}$$

Si los datos se encuentran equiespaciados podemos utilizar las expresiones que analizamos anteriormente

# Aplicación

**Un paracaidista se arroja de un avión y mediante un dispositivo de medición se almacena su ubicación respecto del suelo con un intervalo de tiempo de dos segundos. Luego de procesar los datos se obtiene la siguiente tabla:**

| Tiempo (seg) | Distancia (m) |
|---|---|
| 2 | 17.41 |
| 4 | 62.27 |
| 6 | 126.17 |
| 8 | 203.24 |
| 10 | 289.44 |
| 12 | 381.96 |
| 14 | 478.86 |
| 16 | 578.80 |
| 18 | 680.84 |
| 20 | 784.34 |
| 22 | 888.85 |
| 24 | 994.05 |
| 26 | 1099.74 |
| 28 | 1205.76 |
| 30 | 1312.02 |
| 32 | 1418.44 |
| 34 | 1524.97 |
| 36 | 1631.58 |
| 38 | 1738.24 |
| 40 | 1844.93 |

# Aplicación



## Averiguar su velocidad y aceleración instantánea

| Tiempo (seg) | Distancia (m) | Velocidad (m/seg) | Aceleracion(m/seg$^2$) |
|---|---|---|---|
| 2 | 17.41 | | |
| 4 | 62.27 | | |
| 6 | 126.17 | | |
| 8 | 203.24 | | |
| 10 | 289.44 | | |
| 12 | 381.96 | | |
| 14 | 478.86 | | |
| 16 | 578.80 | | |
| 18 | 680.84 | | |
| 20 | 784.34 | | |
| 22 | 888.85 | | |
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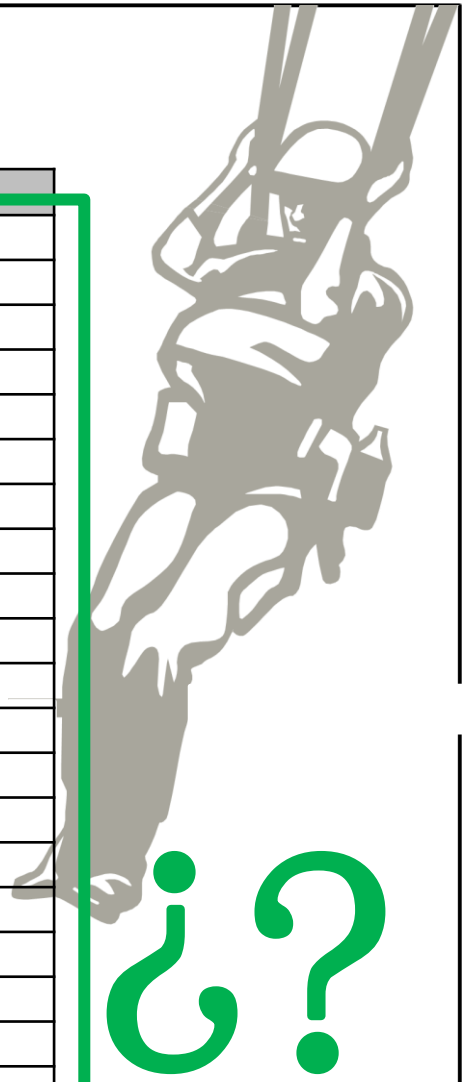
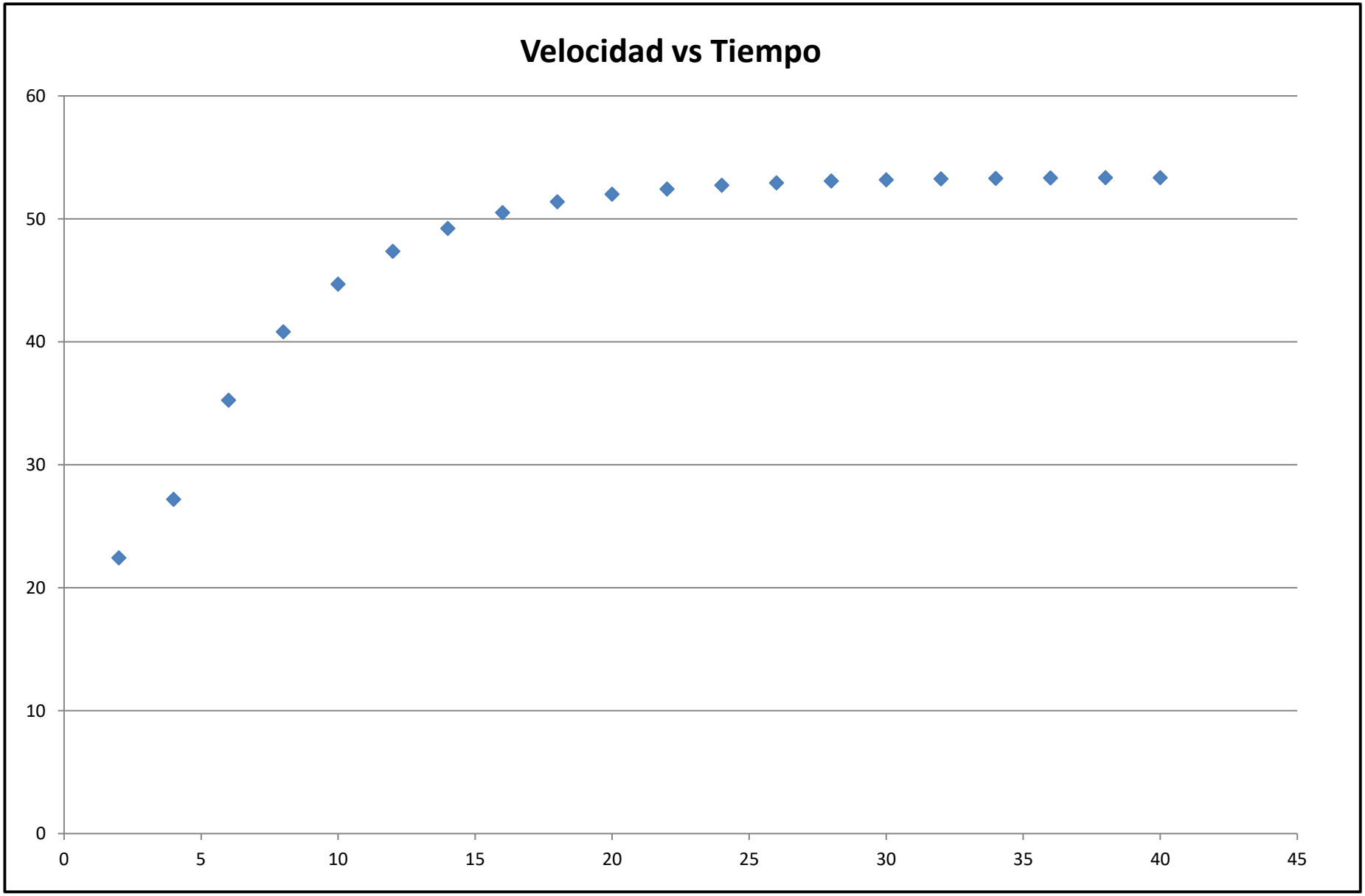
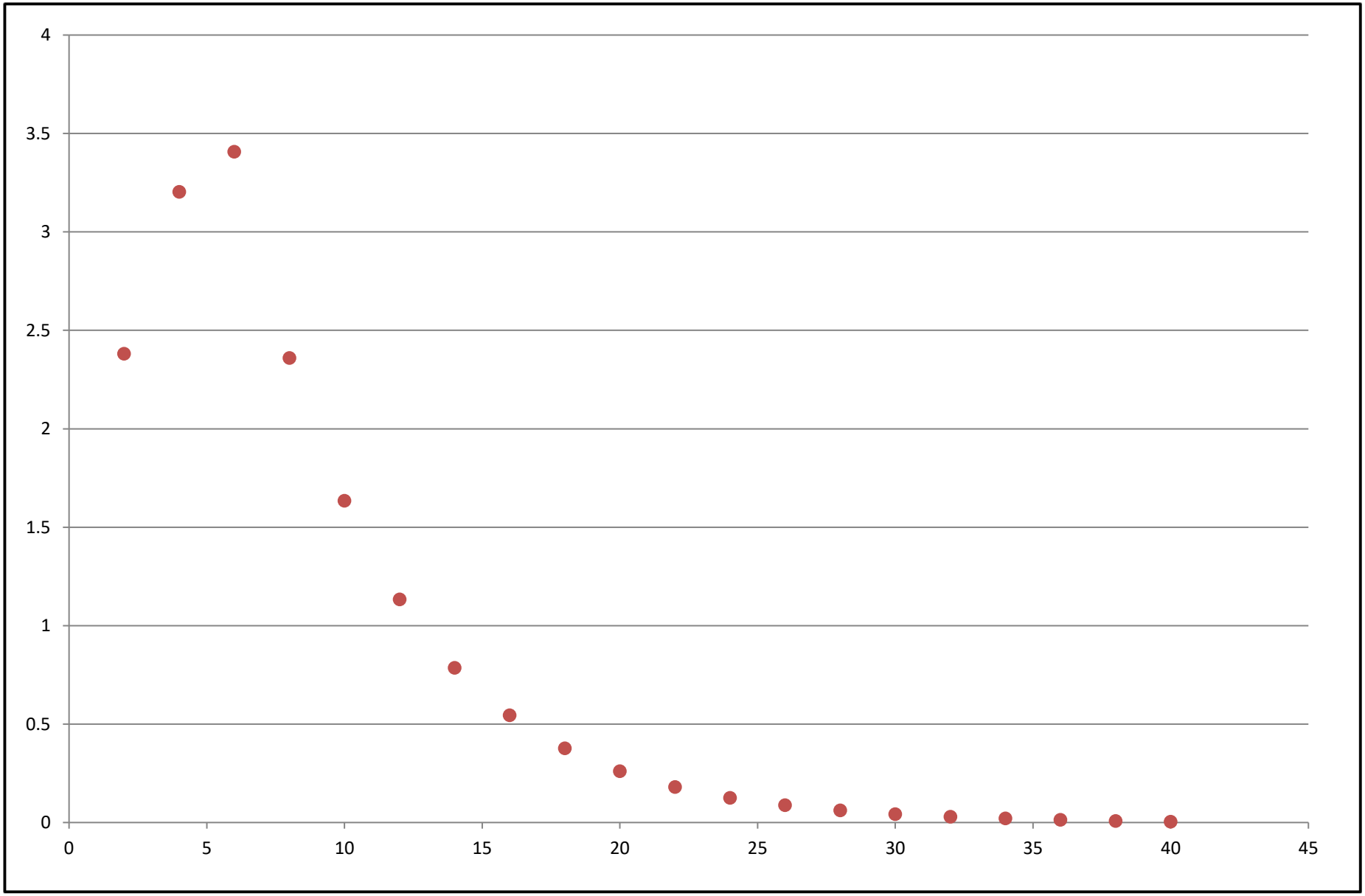
| 26 | 1099.74 | | |
| 28 | 1205.76 | | |
| 30 | 1312.02 | | |
| 32 | 1418.44 | | |
| 34 | 1524.97 | | |
| 36 | 1631.58 | | |
| 38 | 1738.24 | | |
| 40 | 1844.93 | | |



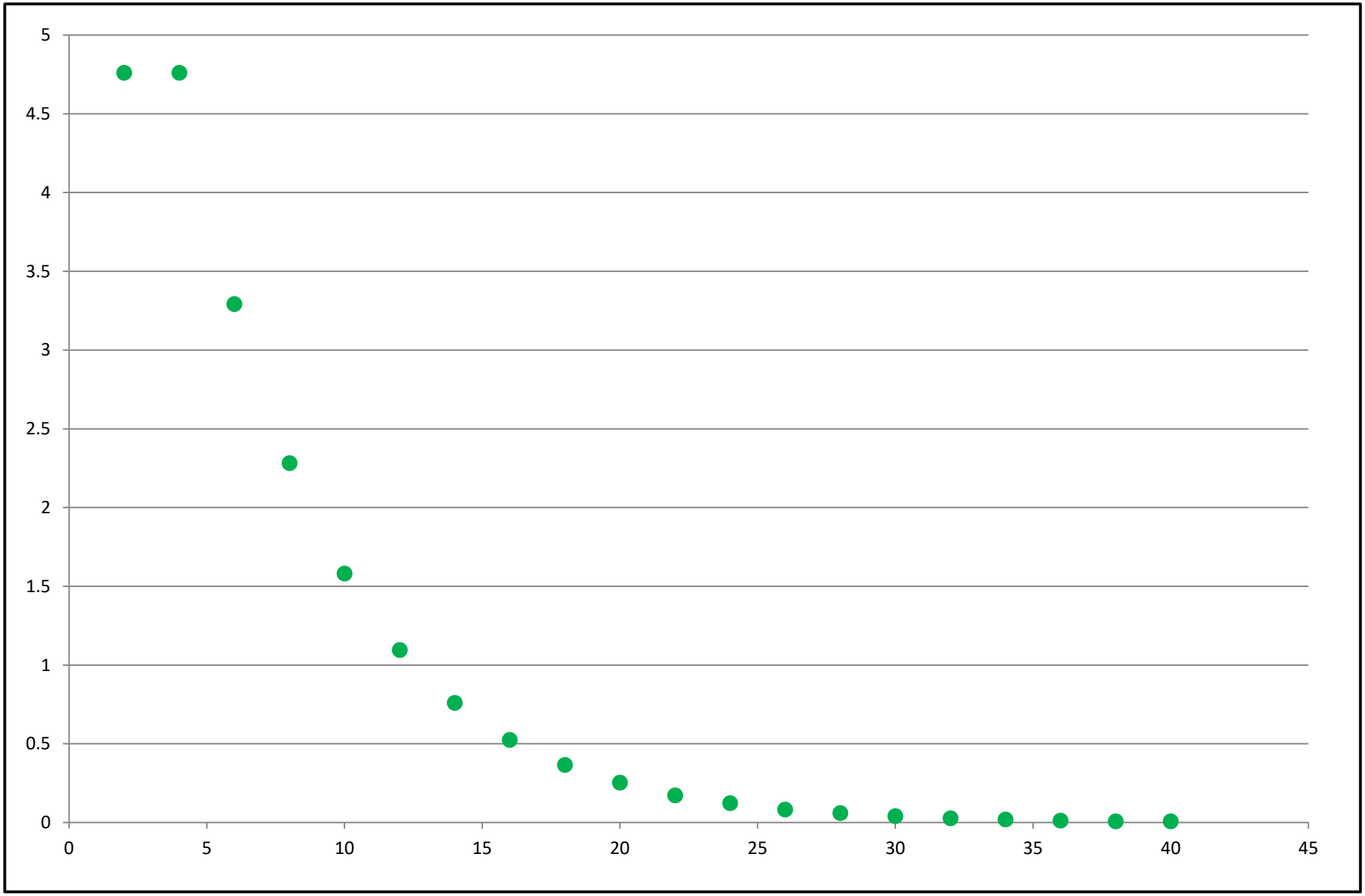
# Velocidad (m/seg)



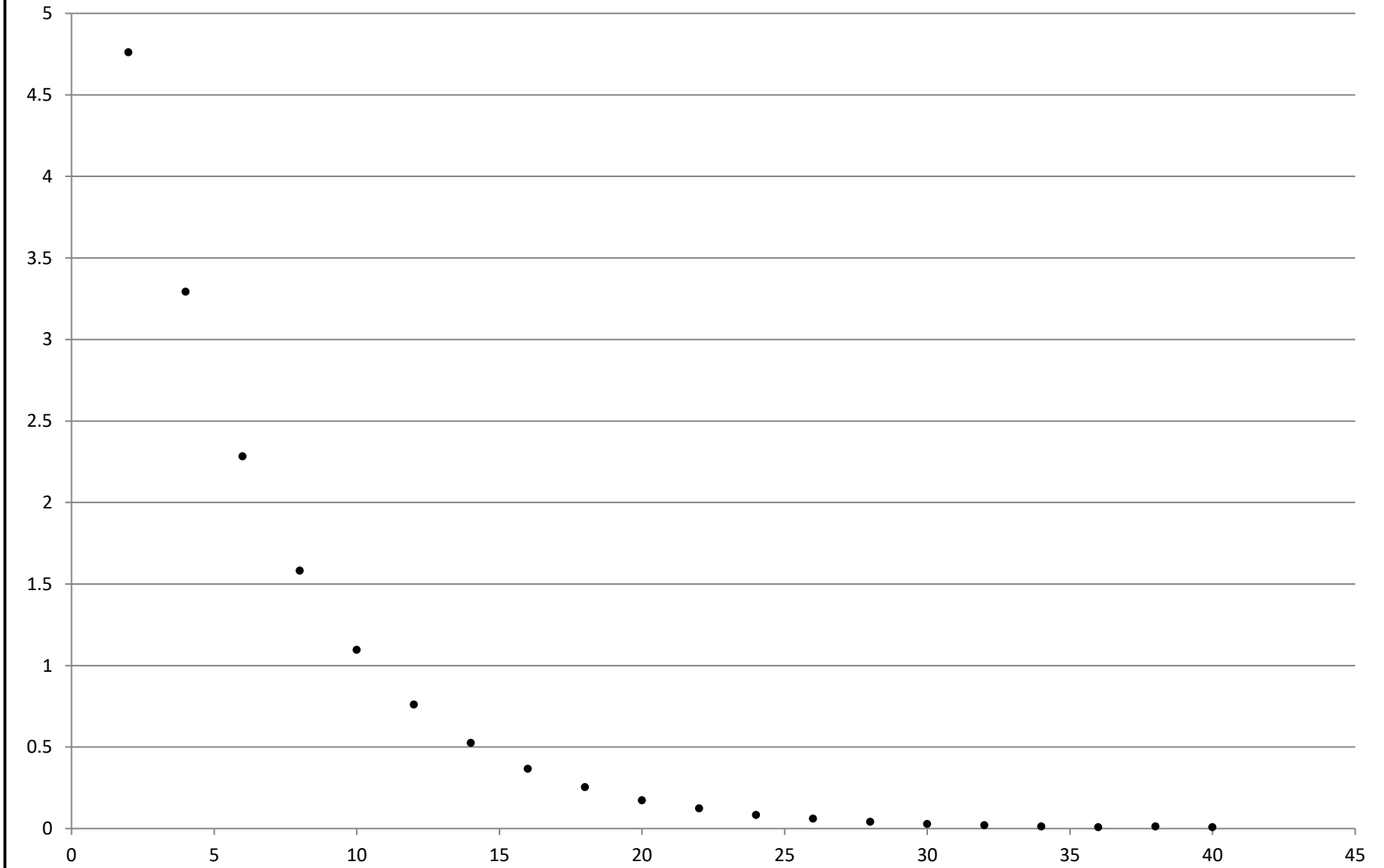
# Aceleración (m/seg$^2$)



# Aceleración (m/seg$^2$)



# Aceleración (m/seg$^2$)



# Aceleración (m/seg$^2$)