

Método de Thomas

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- El método de Thomas se utiliza para la resolución de sistemas de ecuaciones algebraicas lineales cuya matriz de coeficientes resulta tridiagonal.

$$\begin{bmatrix}
 B_1 & C_1 & 0 & 0 & \dots & 0 & 0 & 0 & 0 \\
 A_2 & B_2 & C_2 & 0 & \dots & 0 & 0 & 0 & 0 \\
 0 & A_3 & B_3 & C_3 & \dots & 0 & 0 & 0 & 0 \\
 \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\
 0 & 0 & 0 & 0 & \dots & 0 & A_{n-1} & B_{n-1} & C_{n-1} \\
 0 & 0 & 0 & 0 & \dots & 0 & 0 & A_n & B_n
 \end{bmatrix}
 \begin{bmatrix}
 x_1 \\
 x_2 \\
 x_3 \\
 \vdots \\
 x_{n-1} \\
 x_n
 \end{bmatrix}
 =
 \begin{bmatrix}
 b_1 \\
 b_2 \\
 b_3 \\
 \vdots \\
 b_{n-1} \\
 b_n
 \end{bmatrix}$$

Matriz Tridiagonal

$$\begin{bmatrix}
 B_1 & C_1 & 0 & 0 & \dots & 0 & 0 & 0 & 0 \\
 A_2 & B_2 & C_2 & 0 & \dots & 0 & 0 & 0 & 0 \\
 0 & A_3 & B_3 & C_3 & \dots & 0 & 0 & 0 & 0 \\
 \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\
 0 & 0 & 0 & 0 & \dots & 0 & A_{n-1} & B_{n-1} & C_{n-1} \\
 0 & 0 & 0 & 0 & \dots & 0 & 0 & A_n & B_n
 \end{bmatrix}
 \begin{bmatrix}
 x_1 \\
 x_2 \\
 x_3 \\
 \vdots \\
 x_{n-1} \\
 x_n
 \end{bmatrix}
 =
 \begin{bmatrix}
 b_1 \\
 b_2 \\
 b_3 \\
 \vdots \\
 b_{n-1} \\
 b_n
 \end{bmatrix}$$

Fila 1: $B_1 x_1 + C_1 x_2 = b_1$

$$B_1 x_1 + C_1 x_2 = b_1$$

$$x_1 = \frac{b_1}{B_1} - \frac{C_1}{B_1} x_2$$

$$Q_1 = \frac{b_1}{B_1} \quad \text{y} \quad P_1 = \frac{C_1}{B_1} \quad \Rightarrow \quad x_1 = Q_1 - P_1 x_2$$

$$\begin{bmatrix}
 B_1 & C_1 & 0 & 0 & \dots & 0 & 0 & 0 & 0 \\
 A_2 & B_2 & C_2 & 0 & \dots & 0 & 0 & 0 & 0 \\
 0 & A_3 & B_3 & C_3 & \dots & 0 & 0 & 0 & 0 \\
 \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\
 0 & 0 & 0 & 0 & \dots & 0 & A_{n-1} & B_{n-1} & C_{n-1} \\
 0 & 0 & 0 & 0 & \dots & 0 & 0 & A_n & B_n
 \end{bmatrix}
 \begin{bmatrix}
 x_1 \\
 x_2 \\
 x_3 \\
 \vdots \\
 x_{n-1} \\
 x_n
 \end{bmatrix}
 =
 \begin{bmatrix}
 b_1 \\
 b_2 \\
 b_3 \\
 \vdots \\
 b_{n-1} \\
 b_n
 \end{bmatrix}$$

Fila 2: $A_2 x_1 + B_2 x_2 + C_2 x_3 = b_2$

$$A_2 x_1 + B_2 x_2 + C_2 x_3 = b_2$$

$$x_1 = Q_1 - P_1 x_2 \quad \longrightarrow \quad \text{de la Fila 1}$$

$$A_2(Q_1 - P_1 x_2) + B_2 x_2 + C_2 x_3 = b_2$$

$$A_2 Q_1 - A_2 P_1 x_2 + B_2 x_2 + C_2 x_3 = b_2$$

$$A_2 Q_1 + (B_2 - A_2 P_1) x_2 + C_2 x_3 = b_2 \Rightarrow x_2 = \frac{b_2 - A_2 Q_1}{B_2 - A_2 P_1} - \frac{C_2}{B_2 - A_2 P_1} x_3$$

$$Q_2 = \frac{b_2 - A_2 Q_1}{B_2 - A_2 P_1} \quad \text{y} \quad P_2 = \frac{C_2}{B_2 - A_2 P_1} \Rightarrow x_2 = Q_2 - P_2 x_3$$

$$\begin{bmatrix}
 B_1 & C_1 & 0 & 0 & \dots & 0 & 0 & 0 & 0 \\
 A_2 & B_2 & C_2 & 0 & \dots & 0 & 0 & 0 & 0 \\
 0 & A_3 & B_3 & C_3 & \dots & 0 & 0 & 0 & 0 \\
 \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\
 0 & 0 & 0 & 0 & \dots & 0 & A_{n-1} & B_{n-1} & C_{n-1} \\
 0 & 0 & 0 & 0 & \dots & 0 & 0 & A_n & B_n
 \end{bmatrix}
 \begin{bmatrix}
 x_1 \\
 x_2 \\
 x_3 \\
 \vdots \\
 x_{n-1} \\
 x_n
 \end{bmatrix}
 =
 \begin{bmatrix}
 b_1 \\
 b_2 \\
 b_3 \\
 \vdots \\
 b_{n-1} \\
 b_n
 \end{bmatrix}$$

Fila 3: $A_3x_2 + B_3x_3 + C_3x_4 = b_3$

$$A_3 x_2 + B_3 x_3 + C_3 x_4 = b_3$$

$$x_2 = Q_2 - P_2 x_3 \text{ (de la Fila 2)}$$

$$A_3 (Q_2 - P_2 x_3) + B_3 x_3 + C_3 x_4 = b_3$$

$$A_3 Q_2 - A_3 P_2 x_3 + B_3 x_3 + C_3 x_4 = b_3$$

$$A_3 Q_2 + (B_3 - A_3 P_2) x_3 + C_3 x_4 = b_3 \Rightarrow x_3 = \frac{b_3 - A_3 Q_2}{B_3 - A_3 P_2} - \frac{C_3}{B_3 - A_3 P_2} x_4$$

$$x_3 = Q_3 - P_3 x_4$$

$$Q_2 = \frac{b_2 - A_2 Q_1}{B_2 - A_2 P_1}$$



$$Q_3 = \frac{b_3 - A_3 Q_2}{B_3 - A_3 P_2}$$



$$Q_i = \frac{b_i - A_i Q_{i-1}}{B_i - A_i P_{i-1}}$$

$$P_2 = \frac{C_2}{B_2 - A_2 P_1}$$



$$P_3 = \frac{C_3}{B_3 - A_3 P_2}$$



$$P_i = \frac{C_i}{B_i - A_i P_{i-1}}$$

$$x_2 = Q_2 - P_2 x_3$$



$$x_3 = Q_3 - P_3 x_4$$



$$x_i = Q_i - P_i x_{i+1}$$

Hasta la fila n-1

$$\begin{bmatrix}
 B_1 & C_1 & 0 & 0 & \dots & 0 & 0 & 0 & 0 \\
 A_2 & B_2 & C_2 & 0 & \dots & 0 & 0 & 0 & 0 \\
 0 & A_3 & B_3 & C_3 & \dots & 0 & 0 & 0 & 0 \\
 \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\
 0 & 0 & 0 & 0 & \dots & 0 & A_{n-1} & B_{n-1} & C_{n-1} \\
 0 & 0 & 0 & 0 & \dots & 0 & 0 & A_n & B_n
 \end{bmatrix}
 \begin{bmatrix}
 x_1 \\
 x_2 \\
 x_3 \\
 \vdots \\
 x_{n-1} \\
 x_n
 \end{bmatrix}
 =
 \begin{bmatrix}
 b_1 \\
 b_2 \\
 b_3 \\
 \vdots \\
 b_{n-1} \\
 b_n
 \end{bmatrix}$$

Fila n: $A_n x_{n-1} + B_n x_n = b_n$

$$A_n x_{n-1} + B_n x_n = b_n$$

$$x_{n-1} = Q_{n-1} - P_{n-1} x_n \quad (\text{de la fila "n-1"})$$

$$\left. \begin{aligned} A_n(Q_{n-1} - P_{n-1}x_n) + B_n x_n &= b_n \\ A_n Q_{n-1} - A_n P_{n-1} x_n + B_n x_n &= b_n \\ A_n Q_{n-1} + (B_n - A_n P_{n-1}) x_n &= b_n \end{aligned} \right\} x_n = \frac{b_n - A_n Q_{n-1}}{B_n - A_n P_{n-1}} = Q_n$$

$$P_1 = \frac{C_1}{B_1} \quad y \quad Q_1 = \frac{b_1}{B_1}$$

$$P_i = \frac{C_i}{B_i - A_i P_{i-1}} \quad y \quad Q_i = \frac{b_i - A_i Q_{i-1}}{B_i - A_i P_{i-1}}$$

$$Q_n = \frac{b_n - A_n Q_{n-1}}{B_n - A_n P_{n-1}} \quad \longrightarrow \quad x_n = Q_n$$

$$\begin{bmatrix} B_1 & C_1 & 0 & 0 & \dots & 0 & 0 & 0 & 0 \\ A_2 & B_2 & C_2 & 0 & \dots & 0 & 0 & 0 & 0 \\ 0 & A_3 & B_3 & C_3 & \dots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \dots & 0 & A_{n-1} & B_{n-1} & C_{n-1} \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 & A_n & B_n \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_{n-1} \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_{n-1} \\ b_n \end{bmatrix}$$

$$x_i = Q_i - P_i x_{i+1} \longrightarrow \text{Para cualquier fila}$$

$$x_{n-1} = Q_{n-1} - P_{n-1} x_{n-1+1} \longrightarrow \text{Fila (n-1)}$$

$$x_{n-1} = Q_{n-1} - P_{n-1} x_n$$

¿Que conozco?

Ej: Aplicación en un sistema 4x4

$$\begin{bmatrix} B_1 & C_1 & 0 & 0 \\ A_2 & B_2 & C_2 & 0 \\ 0 & A_3 & B_3 & C_3 \\ 0 & 0 & A_4 & B_4 \end{bmatrix} \times \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}$$

$$P_1 = \frac{C_1}{B_1} \quad Q_1 = \frac{b_1}{B_1} \quad \rightarrow \quad P_2 = \frac{C_2}{B_2 - A_2 P_1} \quad Q_2 = \frac{b_2 - A_2 Q_1}{B_2 - A_2 P_1} \quad \rightarrow \quad P_3 = \frac{C_3}{B_3 - A_3 P_2} \quad Q_3 = \frac{b_3 - A_3 Q_2}{B_3 - A_3 P_2} \quad \rightarrow \quad Q_4 = \frac{b_4 - A_4 Q_3}{B_4 - A_4 P_3}$$

$$x_1 = Q_1 - P_1 x_2 \quad \leftarrow \quad x_2 = Q_2 - P_2 x_3 \quad \leftarrow \quad x_3 = Q_3 - P_3 x_4 \quad \leftarrow \quad x_4 = Q_4$$

$$\begin{array}{ccccccc}
 P_1 = \frac{C_1}{B_1} & \rightarrow & P_2 = \frac{C_2}{B_2 - A_2 P_1} & \rightarrow & P_3 = \frac{C_3}{B_3 - A_3 P_2} & \rightarrow & Q_4 = \frac{b_4 - A_4 Q_3}{B_4 - A_4 P_3} \\
 Q_1 = \frac{b_1}{B_1} & & Q_2 = \frac{b_2 - A_2 Q_1}{B_2 - A_2 P_1} & & Q_3 = \frac{b_3 - A_3 Q_2}{B_3 - A_3 P_2} & & \\
 & & & & & & \downarrow \\
 x_1 = Q_1 - P_1 x_2 & \leftarrow & x_2 = Q_2 - P_2 x_3 & \leftarrow & x_3 = Q_3 - P_3 x_4 & \leftarrow & x_4 = Q_4
 \end{array}$$

$$i = 2:(n-1)$$

$$P_1 = \frac{C_1}{B_1}$$

$$Q_1 = \frac{b_1}{B_1}$$

$$P_i = \frac{C_i}{B_i - A_i P_{i-1}}$$

$$Q_i = \frac{b_i - A_i Q_{i-1}}{B_i - A_i P_{i-1}}$$

$$Q_n = \frac{b_n - A_n Q_{n-1}}{B_n - A_n P_{n-1}}$$

$$x_n = Q_n$$

$$i = (n-1):-1:1$$

$$x_i = Q_i - P_i x_{i+1}$$

```
function x=Thomas(M,b)
```

```
A=diag(M,-1);
```

```
B=diag(M,0);
```

```
C=diag(M,1);
```

```
//conviene agregar ceros en A y C
```

```
A=[0;A];
```

```
C=[C;0];
```

```
n=length(b);
```

```
for i=1:n-1
```

```
    P(i)=C(i)/B(i)-A(i)*P(i-1);
    Q(i)=(b(i)-A(i)*Q(i-1))/(B(i)-A(i)*P(i-1));
endfor
```

```
x(n)=Q(n);
```

```
for i=n-1:-1:1
```

```
    x(i)=Q(i)-P(i)*x(i+1);
endfor
```

```
endfunction
```

$$P_1 = \frac{C_1}{B_1}$$

$$Q_1 = \frac{b_1}{B_1}$$

$$P_i = \frac{C_i}{B_i - A_i P_{i-1}}$$

$$Q_i = \frac{b_i - A_i Q_{i-1}}{B_i - A_i P_{i-1}}$$

$i = 2:(n-1)$

$$Q_n = \frac{b_n - A_n Q_{n-1}}{B_n - A_n P_{n-1}}$$

$$x_n = Q_n$$

$$x_i = Q_i - P_i x_{i+1} \quad i = (n-1):-1:1$$