Sistemas de Ecuaciones No Lineales

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Introducción

Podemos decir que los sistemas de ecuaciones no-lineales son un conjunto de ecuaciones no-lineales que deben satisfacerse en simultaneo.

$$
\begin{aligned}
&f_1\left(x_1, x_2, \ldots, x_n\right) = 0 \\
&f_2\left(x_1, x_2, \ldots, x_n\right) = 0 \\
&\vdots \\
&f_m\left(x_1, x_2, \ldots, x_n\right) = 0
\end{aligned}
$$

$$
\begin{aligned}
f_1(x_1, x_2, \ldots, x_n) &= 0 \\
f_2(x_1, x_2, \ldots, x_n) &= 0 \\
&\vdots \\
f_m(x_1, x_2, \ldots, x_n) &= 0
\end{aligned}
$$

$x_i \in R \qquad \forall\, i = 1, 2, \ldots, n$

$f_j : R^n \to R \qquad \forall\, j = 1, 2, \ldots, m$

n-m: grados de libertad

Sin perdida de generalidad supondremos $m=n$

$$
\begin{aligned}
f_1(x_1, x_2, \ldots, x_n) &= 0 \\
f_2(x_1, x_2, \ldots, x_n) &= 0 \\
&\vdots \\
f_n(x_1, x_2, \ldots, x_n) &= 0
\end{aligned}
$$

Se define la función vectorial $\underline{f}$ asociada al sistema de ecuaciones original y el vector de incógnitas $\underline{x}$:

$$\underline{f}\left(\underline{x}\right)=\begin{bmatrix} f_1\left(x_1,x_2,\ldots,x_n\right) \\ f_2\left(x_1,x_2,\ldots,x_n\right) \\ \vdots \\ f_n\left(x_1,x_2,\ldots,x_n\right) \end{bmatrix} \qquad \underline{x}=\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

Por lo tanto, la expresión compacta de un sistema de ecuaciones algebraicas no lineal corresponde a:

$$\underline{f}\left(\underline{x}\right)=\underline{0} \qquad \begin{aligned} &\underline{x}\in R^n \\ &\underline{f}:R^n\rightarrow R^n \end{aligned}$$

Aproximaciones sucesivas (multivariable)

Matemática Superior Aplicada

UTN

El valor de arranque o semilla corresponde a un vector: $\underline{x}^{(0)} = \underline{\alpha}^{(0)} = \begin{bmatrix} \alpha_1^{(0)} \\ \alpha_2^{(0)} \\ \vdots \\ \alpha_n^{(0)} \end{bmatrix}$

La primera aproximación se obtiene a partir de la función vectorial $\underline{F}$ asociada al sistema original.

$$\underline{x}^{(1)} = \underline{F}\left(\underline{x}^{(0)}\right)$$

Finalmente se desarrolla el proceso iterativo hasta satisfacer la tolerancia o alcanzar el máximo de iteraciones:

$$\underline{x}^{(k+1)} = \underline{F}\left(\underline{x}^{(k)}\right)$$

$$k = 1,\ 2,\ \ldots,\ k_{\max}$$

$$\left\| \underline{x}^{(k+1)} - \underline{x}^{(k)} \right\| < \varepsilon$$

$$\frac{\left\| \underline{x}^{(k+1)} - \underline{x}^{(k)} \right\|}{\left\| \underline{x}^{(k)} \right\|} < \varepsilon_r$$

Al tratarse de vectores el error corresponde a la norma de la diferencia entre dos aproximaciones sucesivas

Ejemplo Aprox. Sucesivas

$$\begin{cases} x_2 + x_1^{\;2} - x_1 - 0.75 = 0 \\ x_2 + 5x_2x_1 - x_1^{\;2} = 0 \end{cases}$$

$$\underline{f}(\underline{x}) = \begin{bmatrix} x_2 + x_1^{\;2} - x_1 - 0.75 \\ x_2 + 5x_2x_1 - x_1^{\;2} \end{bmatrix} \qquad \underline{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \qquad \underline{f}(\underline{x}) = \underline{0}$$

Sistema equivalente:

$$\underline{F}(\underline{x}) = \begin{bmatrix} x_2 + x_1^{\;2} - 0.75 \\ -5x_2x_1 + x_1^{\;2} \end{bmatrix} \qquad \underline{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \qquad \underline{x} = \underline{F}(\underline{x})$$

Ejemplo Aprox. Sucesivas

$$\underline{F}(\underline{x}) = \begin{bmatrix} x_2 + x_1^2 - 0.75 \\ -5x_2x_1 + x_1^2 \end{bmatrix} \quad \underline{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\underline{x}^{(0)} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \xrightarrow{\underline{F}\left(\underline{x}^{(0)}\right)} \underline{x}^{(1)} = \begin{bmatrix} 1.2500000 \\ -4.0000000 \end{bmatrix} \xrightarrow{\underline{F}\left(\underline{x}^{(1)}\right)} \underline{x}^{(2)} = \begin{bmatrix} -3.1875000 \\ 26.5625000 \end{bmatrix}$$

$$\underline{e} = \left\| \underline{x}^{(2)} - \underline{x}^{(1)} \right\| = 30.8829696$$

El error de la iteración 5 corresponde a:

$$\left\| \underline{e} \right\| = 9.6510395 \times 10^{15}$$

¡El sistema no converge!

Ejemplo Aprox. Sucesivas

$$\underline{f}(\underline{x}) = \begin{bmatrix} x_2 + x_1^{\,2} - x_1 - 0.75 \\ x_2 + 5x_2x_1 - x_1^{\,2} \end{bmatrix} \rightarrow \underline{F}(\underline{x}) = \begin{bmatrix} \sqrt{-x_2 + x_1 + 0.75} \\ \left(-x_2 + x_1^{\,2}\right) / \left(5x_1\right) \end{bmatrix}$$

$$\underline{x}^{(0)} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \xrightarrow{\underline{F}\left(\underline{x}^{(0)}\right)} \underline{x}^{(1)} = \begin{bmatrix} 0.8660254 \\ 0 \end{bmatrix} \xrightarrow{\underline{F}\left(\underline{x}^{(1)}\right)} \underline{x}^{(2)} = \begin{bmatrix} 1.2712299 \\ 0.1732051 \end{bmatrix}$$

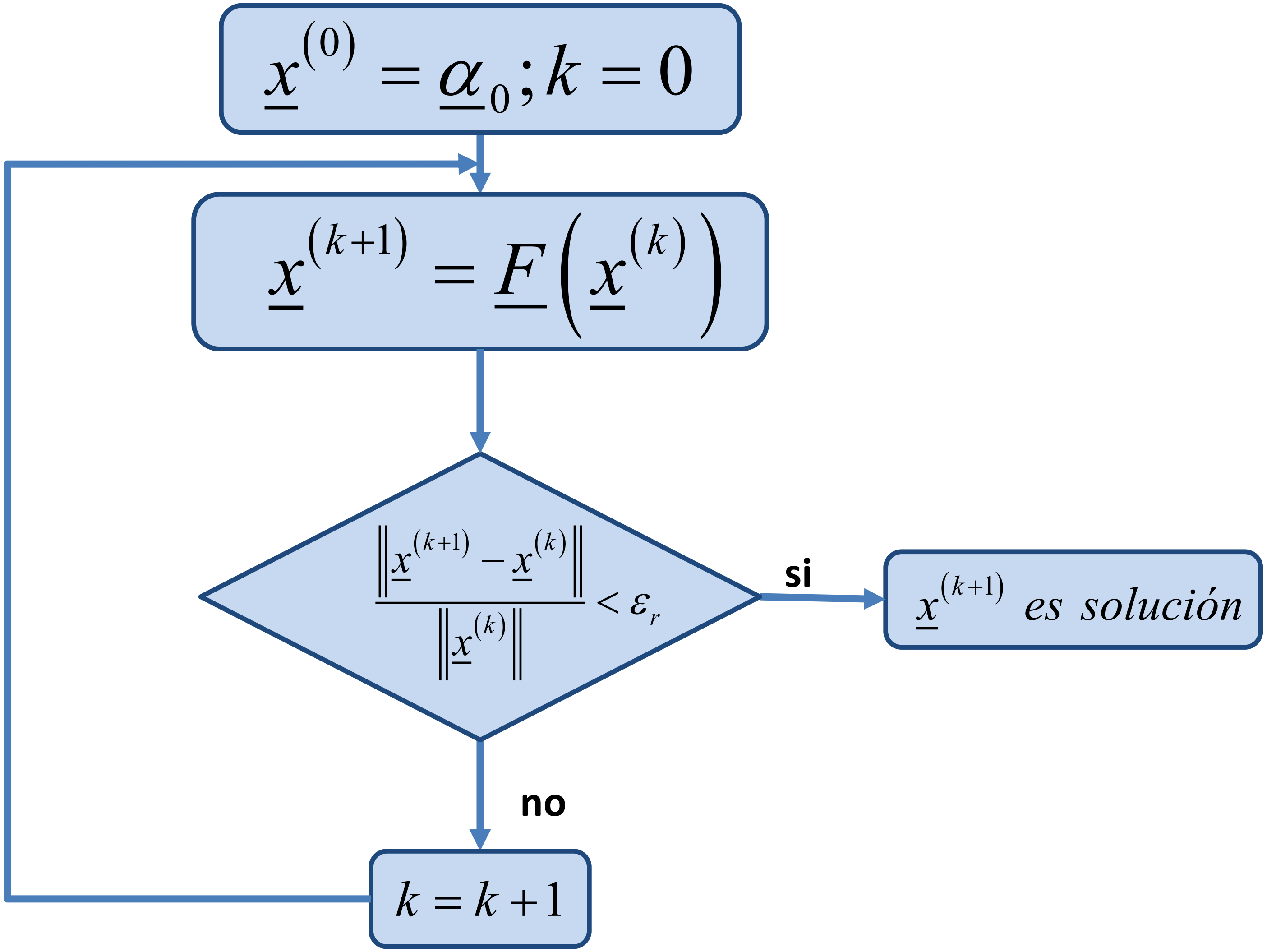
$$\underline{x}^{(9)} = \begin{bmatrix} 1.3720650 \\ 0.2395017 \end{bmatrix} \rightarrow \underline{e} = \left\| \underline{x}^{(9)} - \underline{x}^{(8)} \right\| = 6.0051938 \times 10^{-7}$$

$$F\left(\underline{x}^{(9)}\right) = F\left(\begin{bmatrix} 1.3720650 \\ 0.2395017 \end{bmatrix}\right) = \begin{bmatrix} 1.3720653 \\ 0.2395019 \end{bmatrix}$$

$$f\left(\underline{x}^{(9)}\right) = \begin{bmatrix} -8.4719027 \times 10^{-7} \\ -1.0625308 \times 10^{-6} \end{bmatrix}$$

$$\underline{x}^{*} = \begin{bmatrix} 1.3720650 \\ 0.2395017 \end{bmatrix}$$

Aproximaciones Sucesivas



Aproximaciones Sucesivas (Scilab)

$$\underline{f}(\underline{x}) = \begin{bmatrix} x_2 + x_1^2 - x_1 - 0.75 \\ x_2 + 5x_2x_1 - x_1^2 \end{bmatrix} \rightarrow \underline{F}(\underline{x}) = \begin{bmatrix} \sqrt{-x_2 + x_1 + 0.75} \\ \left(-x_2 + x_1^2\right)/\left(5x_1\right) \end{bmatrix}$$

```
function fx=fsist(x)
   fx = [x(2) + x(1)^2 - x(1)-0.75
       x(2) + 5*x(2)*x(1) - x(1)^2]
endfunction
```

```
function fx=Fsist(x)
   fx = [sqrt(-x(2) + x(1) + 0.75)
       (-x(2) + x(1)^2)/(5*x(1))]
endfunction
```

Aproximaciones Sucesivas (Scilab)


```
function [out,k] =aproxsuc(fun, x0, tol)
x=x0;
for k=1:100
   x(:,k+1)=fun(x(:,k));
   if norm(x(:,k+1)-x(:,k))/norm(x(:,k)) < tol then
      out = x(:,k+1);
      break
   end
end
if k == 100
out=[];
disp('no converge');
end
endfunction
```

Método de Wegstein (multivariable)

Corresponde a una extensión de lo presentado para una variable.

$$\underline{x}^{(0)} = \underline{\alpha}_0$$

$$\underline{x}^{(1)} = \underline{F}\left(\underline{x}^{(0)}\right)$$

$$\underline{\underline{Q}} = \begin{bmatrix} q_1 & 0 & \cdots & 0 \\ 0 & q_2 & \cdots & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & q_n \end{bmatrix}$$

$$\omega_i = \frac{F_i\left(\underline{x}^{(k)}\right) - F_i\left(\underline{x}^{(k-1)}\right)}{x_i^{(k)} - x_i^{(k-1)}} \qquad i = 1 \ a \ n$$

$$q_i = \frac{\omega_i}{\omega_i - 1} \quad i = 1 \ a \ n \rightarrow Q_{ii} = q_i \quad i = 1 \ a \ n$$

$$\underline{x}^{(k+1)} = \underline{\underline{Q}}\,\underline{x}^{(k)} + \left(\underline{\underline{I}} - \underline{\underline{Q}}\right)\underline{F}\left(\underline{x}^{(k)}\right)$$

$$k = 1, 2, 3, \ldots, k_{\max}$$

$$\frac{\left\|\underline{x}^{(k+1)} - \underline{x}^{(k)}\right\|}{\left\|\underline{x}^{(k)}\right\|} < \varepsilon_r$$

$$\left\|\underline{x}^{(k+1)} - \underline{F}\left(\underline{x}^{(k+1)}\right)\right\| < \varepsilon$$

Implementación en el ejemplo

$$\underline{F}\left(\underline{x}\right)=\begin{bmatrix} x_2+x_1{}^2-0.75 \\ -5x_2x_1+x_1{}^2 \end{bmatrix} \qquad \underline{x}=\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\underline{x}^{(0)}=\begin{bmatrix} 1 \\ 1 \end{bmatrix} \xrightarrow{\underline{F}\left(\underline{x}^{(0)}\right)} \underline{x}^{(1)}=\begin{bmatrix} 1.2500000 \\ -4.0000000 \end{bmatrix} \xrightarrow{\underline{F}\left(\underline{x}^{(1)}\right)} \begin{bmatrix} \text{-}3.1875000 \\ 26.5625000 \end{bmatrix}$$

$$\omega_i=\frac{F_i\left(\underline{x}^{(1)}\right)-F_i\left(\underline{x}^{(0)}\right)}{x_i{}^{(1)}-x_i{}^{(0)}} \quad i=1\ a\ 2 \ \rightarrow \underline{\omega}=\begin{bmatrix} -17.75 \\ -6.1125 \end{bmatrix}$$

Implementación en el ejemplo

$$q_i = \frac{\omega_i}{\omega_i - 1} \quad i = 1 \; a \; 2 \quad \rightarrow \underline{\underline{Q}} = \begin{bmatrix} 0.9466667 & 0 \\ 0 & 0.8594025 \end{bmatrix}$$

$$\underline{x}^{(2)} = \underline{\underline{Q}}\,\underline{x}^{(1)} + \left(\underline{\underline{I}} - \underline{\underline{Q}}\right)\underline{F}\left(\underline{x}^{(1)}\right)$$

$$\underline{x}^{(2)} = \begin{bmatrix} 0.9466667 & 0 \\ 0 & 0.8594025 \end{bmatrix}\begin{bmatrix} 1.25 \\ -4 \end{bmatrix} + \begin{bmatrix} 0.0533333 & 0 \\ 0 & 0.1405975 \end{bmatrix}\begin{bmatrix} -3.1875 \\ 26.5625 \end{bmatrix}$$

$$\underline{x}^{(2)} = \begin{bmatrix} 1.0133333 \\ 0.2970123 \end{bmatrix} \rightarrow error = \left\| \underline{x}^{(2)} - \underline{x}^{(1)} \right\| = 4.3035248$$

$$\underline{x}^{(43)} = \begin{bmatrix} 1.3720655 \\ 0.2395020 \end{bmatrix} \rightarrow error = 1.343 \times 10^{-11}$$

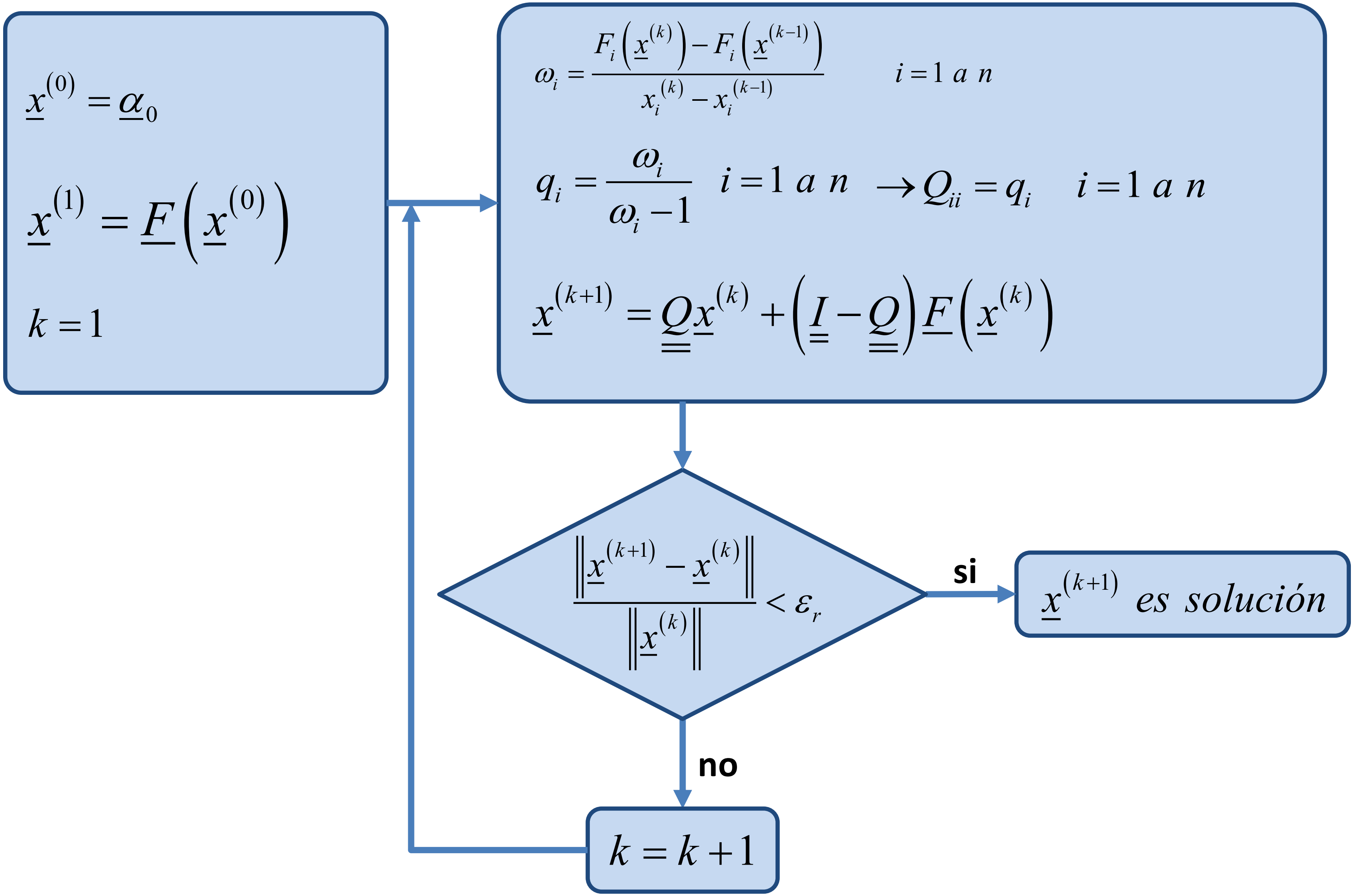
Implementación en el ejemplo

$$F\left(\underline{x}^{(43)}\right) = F\left(\begin{bmatrix} 1.3720655 \\ 0.2395020 \end{bmatrix}\right) = \begin{bmatrix} 1.3720658 \\ 0.2395020 \end{bmatrix}$$

$$f\left(\underline{x}^{(43)}\right) = f\left(\begin{bmatrix} 1.3720655 \\ 0.2395020 \end{bmatrix}\right) = \begin{bmatrix} 2.672907\text{e-}7 \\ -1.921863\text{e-}11 \end{bmatrix}$$

$$\underline{x}^{*} = \begin{bmatrix} 1.3720644 \\ 0.2395041 \end{bmatrix}$$

Método de Wegstein



Método de Wegstein (scilab)

```
function [out,k] =wegstein(fun, x0, tol)
   n=length(x0);
   x(:,1)=x0;
   x(:,2)=fun(x(:,1));
   FF(:,1)=fun(x(:,1));
   FF(:,2)=fun(x(:,2));
   for k=2:100
      w=(FF(:,k)-FF(:,k-1))./(x(:,k)-x(:,k-1));    q=w./(w-1);  Q=diag(q);
      x(:,k+1) = Q*x(:,k)+(eye(n,n)-Q)*FF(:,k)
      if norm(x(:,k+1)-x(:,k))/norm(x(:,k)) < tol then
         out = x(:,k+1);
         break
      end
      FF(:,k+1) = fun(x(:,k+1));
   end
   if k == 100
      out=[];
      disp('no converge');
   end
endfunction
```

Repaso

Serie de Taylor en una variable: $f(x)=\sum_{k=0}^{\infty}\frac{1}{k!}f^{k}(x_0)(x-x_0)^{k}$

$$f(x,x_0)=f(x_0)+f'(x_0)(x-x_0)$$ Taylor de primer orden

Utilizando el valor actual de la variable en un proceso iterativo $x^{(k)}$ se aproxima por Taylor de primer orden el valor de la función en el siguiente punto $x^{(k+1)}$:

$$f\left(x^{(k+1)},x^{(k)}\right)=f\left(x^{(k)}\right)+f'\left(x^{(k)}\right)\left(x^{(k+1)}-x^{(k)}\right)$$

Igualamos a cero para aproximar la raíz: $f\left(x^{(k)}\right)+f'\left(x^{(k)}\right)\left(x^{(k+1)}-x^{(k)}\right)=0$

$$x^{(k+1)}=x^{(k)}-\frac{f\left(x^{(k)}\right)}{f'\left(x^{(k)}\right)}$$

¡Newton-Rhapson!

Serie de Taylor en multivariable: $f\left(\underline{x}\right)=f\left(\underline{x}_0\right)+\sum_{k=1}^{\infty}\frac{1}{k!}d^k f\left(\underline{x}\right)$

Se define el diferencial r de una función f como:

$$d^r f\left(\underline{x}\right)=\underbrace{\sum_{i=1}^{n}\sum_{j=1}^{n}\cdots\sum_{k=1}^{n}}_{r\ \ sumatorias}\left(x-x_0\right)_i\left(x-x_0\right)_j\cdots\left(x-x_0\right)_k\frac{\partial^r f\left(\underline{x}_0\right)}{\partial x_i\partial x_j\cdots\partial x_k}$$

Para $r=1$:

$$df\left(\underline{x}\right)=\sum_{i=1}^{n}\left(x-x_0\right)_i\frac{\partial f\left(\underline{x}_0\right)}{\partial x_i}$$

$$df\left(\underline{x}\right)=\left(x-x_0\right)_1\frac{\partial f\left(\underline{x}_0\right)}{\partial x_1}+\left(x-x_0\right)_2\frac{\partial f\left(\underline{x}_0\right)}{\partial x_2}+\cdots+\left(x-x_0\right)_n\frac{\partial f\left(\underline{x}_0\right)}{\partial x_n}$$

Taylor Multivariable

$$df(\underline{x}) = (x - x_0)_1 \frac{\partial f(\underline{x}_0)}{\partial x_1} + (x - x_0)_2 \frac{\partial f(\underline{x}_0)}{\partial x_2} + \cdots + (x - x_0)_n \frac{\partial f(\underline{x}_0)}{\partial x_n}$$

$$df(\underline{x}) = (\underline{x} - \underline{x}_0)^T \nabla f(\underline{x}_0)$$

$$\begin{matrix} \dfrac{\partial f(\underline{x}_0)}{\partial x_1} \\ \dfrac{\partial f(\underline{x}_0)}{\partial x_2} \\ \vdots \\ \dfrac{\partial f(\underline{x}_0)}{\partial x_n} \end{matrix}$$

$$(x - x_0)_1 \quad (x - x_0)_2 \quad \cdots \quad (x - x_0)_n$$

Taylor Multivariable

$$df\left(\underline{x}\right)=\left(x-x_0\right)_1\frac{\partial f\left(\underline{x}_0\right)}{\partial x_1}+\left(x-x_0\right)_2\frac{\partial f\left(\underline{x}_0\right)}{\partial x_2}+\cdots+\left(x-x_0\right)_n\frac{\partial f\left(\underline{x}_0\right)}{\partial x_n}$$

$$df\left(\underline{x}\right)=\nabla f\left(\underline{x}_0\right)^T\left(\underline{x}-\underline{x}_0\right)$$

$$\left(x-x_0\right)_1$$

$$\left(x-x_0\right)_2$$

$$\vdots$$

$$\left(x-x_0\right)_3$$

$$\frac{\partial f\left(\underline{x}_0\right)}{\partial x_1}\quad\frac{\partial f\left(\underline{x}_0\right)}{\partial x_2}\quad\cdots\quad\frac{\partial f\left(\underline{x}_0\right)}{\partial x_n}$$

Taylor Multivariable

Aproximación de Taylor de primer orden de una función de valor real de n variables:

$$f\left(\underline{x},\underline{x}_0\right)=f\left(\underline{x}_0\right)+\nabla f\left(\underline{x}_0\right)^T\left(\underline{x}-\underline{x}_0\right) \qquad f\left(\underline{x}\right):\mathbb{R}^n\to\mathbb{R}$$

Utilizando el valor actual de las variables $\underline{x}^{(k)}$ se aproxima por Taylor de primer orden el valor de cada una de las ecuaciones del sistema en el siguiente punto $\underline{x}^{(k+1)}$:

$$\underline{f}\left(\underline{x}\right)=\begin{bmatrix} f_1\left(x_1,x_2,\ldots,x_n\right) \\ f_2\left(x_1,x_2,\ldots,x_n\right) \\ \vdots \\ f_n\left(x_1,x_2,\ldots,x_n\right)\end{bmatrix}$$

$$\underline{f}\left(\underline{x}\right):\mathbb{R}^n\to\mathbb{R}^n$$

$$f_i\left(\underline{x}\right):\mathbb{R}^n\to\mathbb{R}$$

$$f_1\left(\underline{x}^{(k+1)},\underline{x}^{(k)}\right)=f_1\left(\underline{x}^{(k)}\right)+\nabla f_1\left(\underline{x}^{(k)}\right)^T\left(\underline{x}^{(k+1)}-\underline{x}^{(k)}\right)$$

$$f_2\left(\underline{x}^{(k+1)},\underline{x}^{(k)}\right)=f_2\left(\underline{x}^{(k)}\right)+\nabla f_2\left(\underline{x}^{(k)}\right)^T\left(\underline{x}^{(k+1)}-\underline{x}^{(k)}\right)$$

$$\vdots$$

$$f_n\left(\underline{x}^{(k+1)},\underline{x}^{(k)}\right)=f_n\left(\underline{x}^{(k)}\right)+\nabla f_n\left(\underline{x}^{(k)}\right)^T\left(\underline{x}^{(k+1)}-\underline{x}^{(k)}\right)$$

Aproximación del SEANL

La aproximación del sistema corresponde a:

$$\underline{\tilde{f}}\left(\underline{x}^{(k+1)},\underline{x}^{(k)}\right)=\begin{bmatrix} f_1\left(\underline{x}^{(k)}\right)+\nabla f_1\left(\underline{x}^{(k)}\right)^T\left(\underline{x}^{(k+1)}-\underline{x}^{(k)}\right) \\ f_2\left(\underline{x}^{(k)}\right)+\nabla f_2\left(\underline{x}^{(k)}\right)^T\left(\underline{x}^{(k+1)}-\underline{x}^{(k)}\right) \\ \vdots \\ f_n\left(\underline{x}^{(k)}\right)+\nabla f_n\left(\underline{x}^{(k)}\right)^T\left(\underline{x}^{(k+1)}-\underline{x}^{(k)}\right) \end{bmatrix}$$

Aproximación del SEANL

$$\underline{\tilde{f}}\left(\underline{x}^{(k+1)},\underline{x}^{(k)}\right)=\begin{bmatrix} f_1\left(\underline{x}^{(k)}\right)+\nabla f_1\left(\underline{x}^{(k)}\right)^T\left(\underline{x}^{(k+1)}-\underline{x}^{(k)}\right) \\ f_2\left(\underline{x}^{(k)}\right)+\nabla f_2\left(\underline{x}^{(k)}\right)^T\left(\underline{x}^{(k+1)}-\underline{x}^{(k)}\right) \\ \vdots \\ f_n\left(\underline{x}^{(k)}\right)+\nabla f_n\left(\underline{x}^{(k)}\right)^T\left(\underline{x}^{(k+1)}-\underline{x}^{(k)}\right) \end{bmatrix}$$

$$\underline{\tilde{f}}\left(\underline{x}^{(k+1)},\underline{x}^{(k)}\right)=\begin{bmatrix} f_1\left(\underline{x}^{(k)}\right) \\ f_2\left(\underline{x}^{(k)}\right) \\ \vdots \\ f_n\left(\underline{x}^{(k)}\right) \end{bmatrix}+\begin{bmatrix} \nabla f_1\left(\underline{x}^{(k)}\right)^T\left(\underline{x}^{(k+1)}-\underline{x}^{(k)}\right) \\ \nabla f_2\left(\underline{x}^{(k)}\right)^T\left(\underline{x}^{(k+1)}-\underline{x}^{(k)}\right) \\ \vdots \\ \nabla f_n\left(\underline{x}^{(k)}\right)^T\left(\underline{x}^{(k+1)}-\underline{x}^{(k)}\right) \end{bmatrix}$$

Aproximación del SEANL

$$\underline{\tilde{f}}\left(\underline{x}^{(k+1)},\underline{x}^{(k)}\right)=\begin{bmatrix} f_1\left(\underline{x}^{(k)}\right) \\ f_2\left(\underline{x}^{(k)}\right) \\ \vdots \\ f_n\left(\underline{x}^{(k)}\right) \end{bmatrix}+\begin{bmatrix} \nabla f_1\left(\underline{x}^{(k)}\right)^T\left(\underline{x}^{(k+1)}-\underline{x}^{(k)}\right) \\ \nabla f_2\left(\underline{x}^{(k)}\right)^T\left(\underline{x}^{(k+1)}-\underline{x}^{(k)}\right) \\ \vdots \\ \nabla f_n\left(\underline{x}^{(k)}\right)^T\left(\underline{x}^{(k+1)}-\underline{x}^{(k)}\right) \end{bmatrix}$$

$$\underline{\tilde{f}}\left(\underline{x}^{(k+1)},\underline{x}^{(k)}\right)=\underline{f}\left(\underline{x}^{(k)}\right)+\begin{bmatrix} \nabla f_1\left(\underline{x}^{(k)}\right)^T \\ \nabla f_2\left(\underline{x}^{(k)}\right)^T \\ \vdots \\ \nabla f_n\left(\underline{x}^{(k)}\right)^T \end{bmatrix}\left(\underline{x}^{(k+1)}-\underline{x}^{(k)}\right)$$

Matriz Jacobiana

Matriz Jacobiana

La matriz jacobina es una función matricial formada por los gradiente transpuestos de cada una de la ecuaciones que conforman el sistema de ecuaciones.

$$\underline{f}(\underline{x}) = \begin{bmatrix} f_1(x_1, x_2, \dots, x_n) \\ f_2(x_1, x_2, \dots, x_n) \\ \vdots \\ f_n(x_1, x_2, \dots, x_n) \end{bmatrix} \rightarrow \underline{\underline{J}}(\underline{x}) = \begin{bmatrix} \dfrac{\partial f_1}{\partial x_1} & \dfrac{\partial f_1}{\partial x_2} & \cdots & \dfrac{\partial f_1}{\partial x_n} \\ \dfrac{\partial f_2}{\partial x_1} & \dfrac{\partial f_2}{\partial x_2} & \cdots & \dfrac{\partial f_2}{\partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \dfrac{\partial f_n}{\partial x_1} & \dfrac{\partial f_n}{\partial x_2} & \cdots & \dfrac{\partial f_n}{\partial x_n} \end{bmatrix}$$

$$\nabla f_1(\underline{x})^T$$

$$\nabla f_2(\underline{x})^T$$

$$\nabla f_n(\underline{x})^T$$

Aproximación del SEANL

Entonces, la aproximación del sistema corresponde a:

$$\underline{\tilde{f}}\left(\underline{x}^{(k+1)},\underline{x}^{(k)}\right)=\underline{f}\left(\underline{x}^{(k)}\right)+\underline{\underline{J}}\left(\underline{x}^{(k)}\right)\left(\underline{x}^{(k+1)}-\underline{x}^{(k)}\right)$$

Igualamos a cero para aproximar la solución:

$$\underline{f}\left(\underline{x}^{(k)}\right)+\underline{\underline{J}}\left(\underline{x}^{(k)}\right)\left(\underline{x}^{(k+1)}-\underline{x}^{(k)}\right)=\underline{0}$$

Finalmente se llega a:

$$\underline{x}^{(k+1)}=\underline{x}^{(k)}-\underline{\underline{J}}\left(\underline{x}^{(k)}\right)^{-1}\underline{f}\left(\underline{x}^{(k)}\right)$$

Formula recursiva del método de newton

La formula recursiva de Newton corresponde a:

$$\underline{x}^{(k+1)} = \underline{x}^{(k)} - \underline{\underline{J}}^{-1}\left(\underline{x}^{(k)}\right)\underline{f}\left(\underline{x}^{(k)}\right)$$

Inversa de la matriz J evaluada en el punto

$$k = 1, 2, \ldots, k_{max}$$

$$\frac{\left\|\underline{x}^{(k+1)} - \underline{x}^{(k)}\right\|}{\left\|\underline{x}^{(k)}\right\|} < \varepsilon_r$$

$$\left\|\underline{f}\left(\underline{x}^{(k+1)}\right)\right\| < \varepsilon$$

$$\underline{\underline{J}} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \cdots & \frac{\partial f_n}{\partial x_n} \end{bmatrix}$$

Matriz Jacobiana de la función f

$$\underline{x} \in R^n$$

$$\underline{f} : R^n \rightarrow R^n$$

$$\underline{\underline{J}} : R^n \rightarrow R^{n \times n}$$

Implementación en el ejemplo

$$\underline{f}\left(\underline{x}\right)=\begin{bmatrix} x_2+x_1{}^2-x_1-0.75 \\ x_2+5x_2x_1-x_1{}^2 \end{bmatrix} \qquad \underline{\underline{J}}\left(\underline{x}\right)=\begin{bmatrix} 2x_1-1 & 1 \\ 5x_2-2x_1 & 1+5x_1 \end{bmatrix}$$

$$\underline{x}^{(0)}=\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

```
function Jx=Jsist(x)
   Jx = [2*x(1)-1          1
         5*x(2)-2*x(1)    1 + 5*x(1)];
endfunction
```

$$\underline{x}^{(1)}=\begin{bmatrix} 1 \\ 1 \end{bmatrix}-\begin{bmatrix} 2 & -1/3 \\ -1 & 1/3 \end{bmatrix}\begin{bmatrix} 0.25 \\ 5 \end{bmatrix}=\begin{bmatrix} 2.1666667 \\ -0.4166667 \end{bmatrix}$$

$$\underline{x}^{(2)}=\begin{bmatrix} 2.1666667 \\ -0.4166667 \end{bmatrix}-\begin{bmatrix} 0.2580254 & -0.0218050 \\ 0.1399152 & 0.0726832 \end{bmatrix}\begin{bmatrix} 1.3611111 \\ -9.6250000 \end{bmatrix}=\begin{bmatrix} 1.6055926 \\ 0.0924692 \end{bmatrix}$$

$$\underline{e}=\underline{x}^{(2)}-\underline{x}^{(1)}=\begin{bmatrix} 1.6055926 \\ 0.0924692 \end{bmatrix}-\begin{bmatrix} 2.1666667 \\ -0.4166667 \end{bmatrix}=\begin{bmatrix} -0.5610741 \\ 0.5091359 \end{bmatrix}\rightarrow\left\|\underline{e}\right\|=0.7576434$$

Implementación en el ejemplo

$$\underline{x}^{(3)} = \begin{bmatrix} 1.4037040 \\ 0.224078 \end{bmatrix} \rightarrow \|\underline{e}\| = 0.2409977$$

$$\underline{x}^{(4)} = \begin{bmatrix} 1.3727742 \\ 0.2392220 \end{bmatrix} \rightarrow \|\underline{e}\| = 0.0344382$$

$$\underline{x}^{(5)} = \begin{bmatrix} 1.3720658 \\ 0.2395018 \end{bmatrix} \rightarrow \|\underline{e}\| = 7.6168800 \times 10^{-4}$$

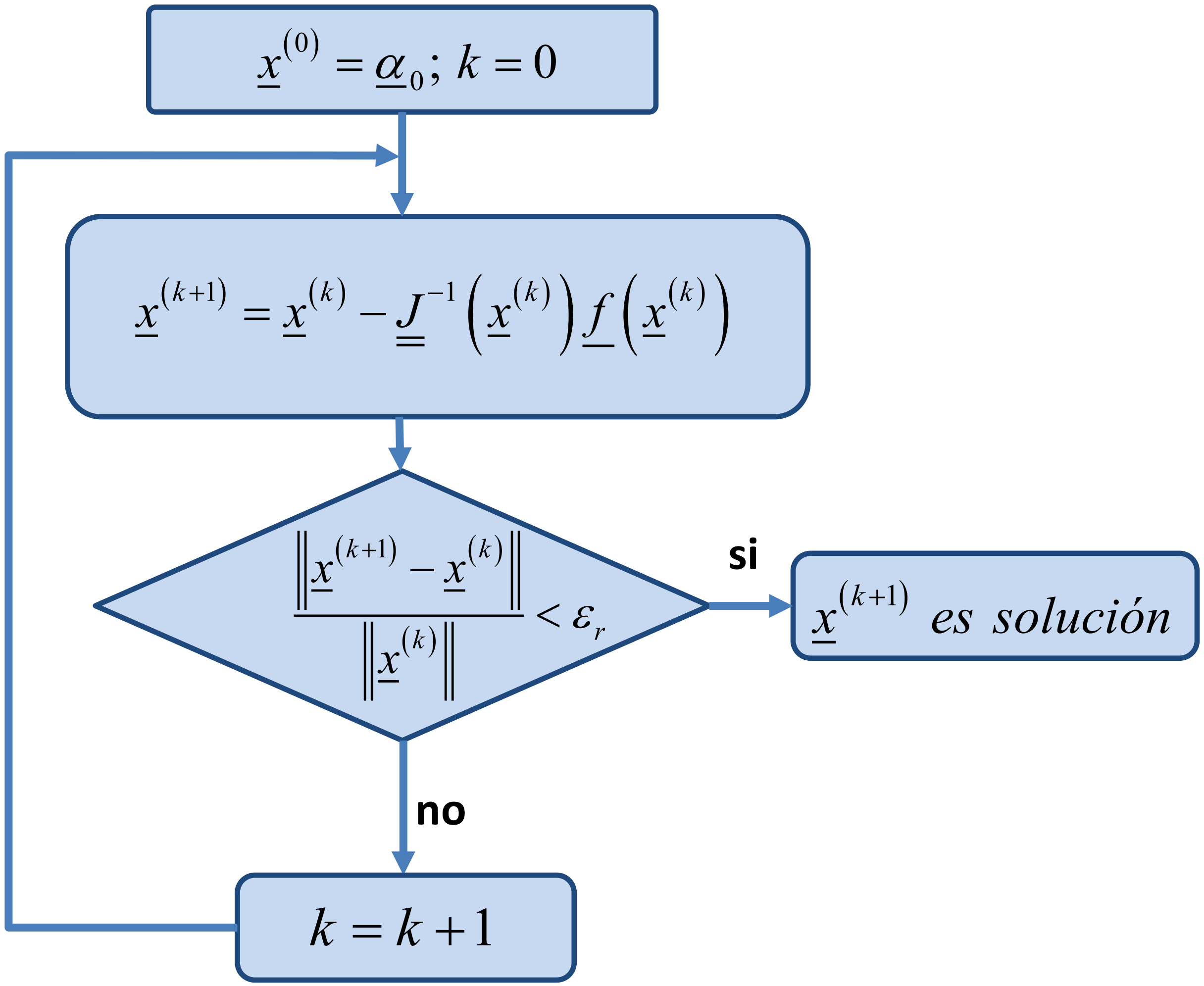
$$\underline{x}^{*} = \begin{bmatrix} 1.3720654 \\ 0.2395019 \end{bmatrix}$$

Podemos afirmar que es solución del sistema.

El error es del orden de 10^{-7}

$$\underline{f}\left(\underline{x}^{*}\right) = \begin{bmatrix} 0.1267875 \times 10^{-12} \\ -0.3406164 \times 10^{-12} \end{bmatrix}$$

Método de Newton



Método de Newton (scilab)

```
function [out,k] =newton(fun, Jx, x0, tol)
    x=x0;
    for k=1:100
        x(:,k+1)=x(:,k) - inv(Jx(x(:,k)))*fun(x(:,k));
        if norm(x(:,k+1)-x(:,k))/norm(x(:,k)) < tol then
            out = x(:,k+1);
            break
        end
    end
    if k == 100
        out=[];
        disp('no converge');
    end
endfunction
```

Método de Newton (II)

La formula recursiva de Newton corresponde a:

$$\underline{x}^{(k+1)} = \underline{x}^{(k)} - \underline{\underline{J}}^{-1}\left(\underline{x}^{(k)}\right)\underline{f}\left(\underline{x}^{(k)}\right)$$

Es la solución a un SEAL

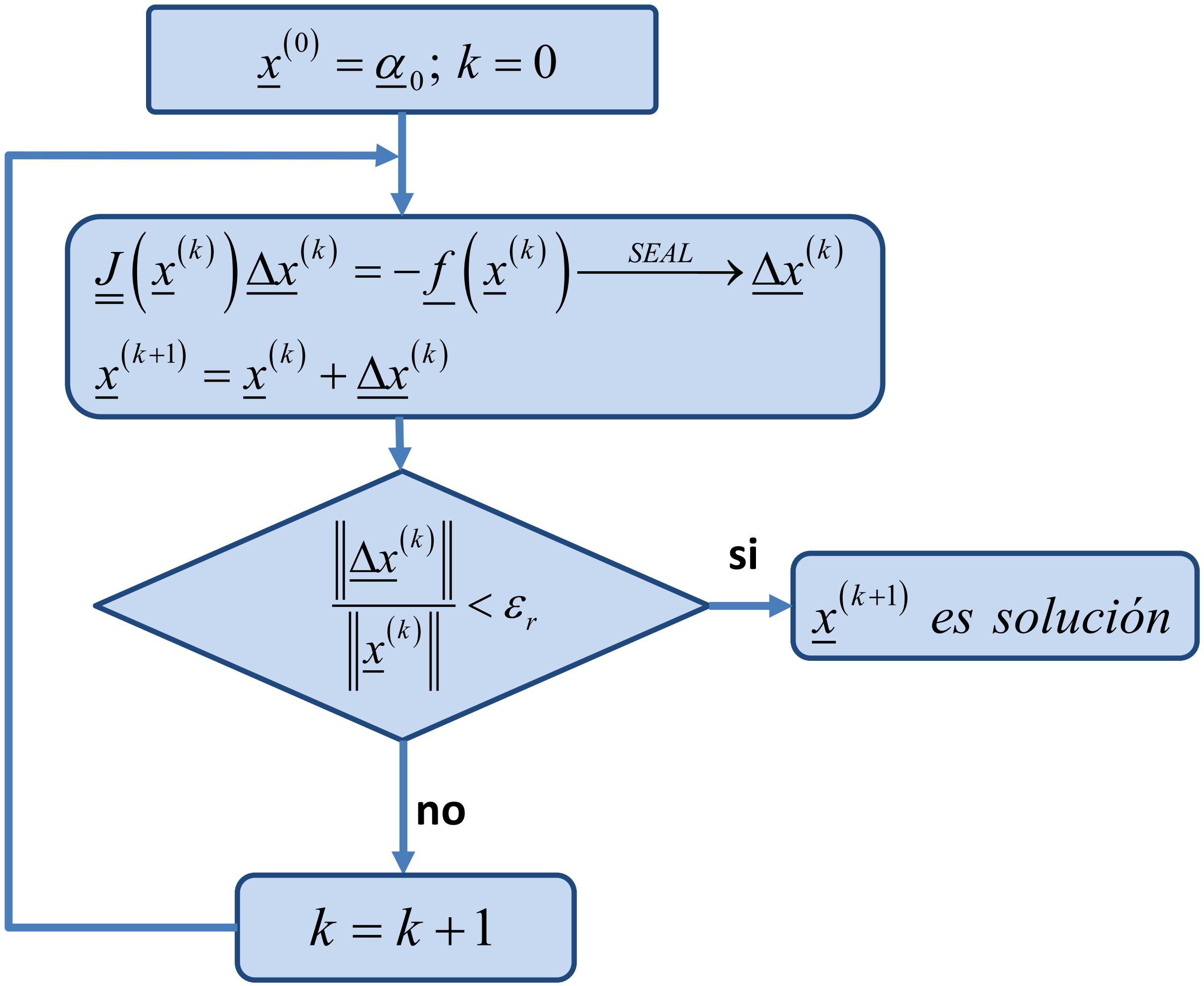
$$\underline{\underline{J}}\left(\underline{x}^{(k)}\right)\underline{\Delta x}^{(k)} = -\underline{f}\left(\underline{x}^{(k)}\right)$$

$$\underline{x}^{(k+1)} = \underline{x}^{(k)} + \underline{\Delta x}^{(k)}$$

Matemática Superior Aplicada

Método de Newton (II)

UTN



Método de Newton (II - scilab)

```
function [out,k] =newtonII(fun, Jx, x0, tol)
    x=x0;
    for k=1:100
        dx = Jx(x(:,k))\(-fun(x(:,k)));
        x(:,k+1)=x(:,k) + dx;
        if norm(dx)/norm(x(:,k)) < tol then
            out = x(:,k+1);
            break
        end
    end
    if k == 100
        out=[];
        disp('no converge');
    end
endfunction
```

Método de Newton con factor de relajación

Variación de la solución en la iteración actual (Δx):

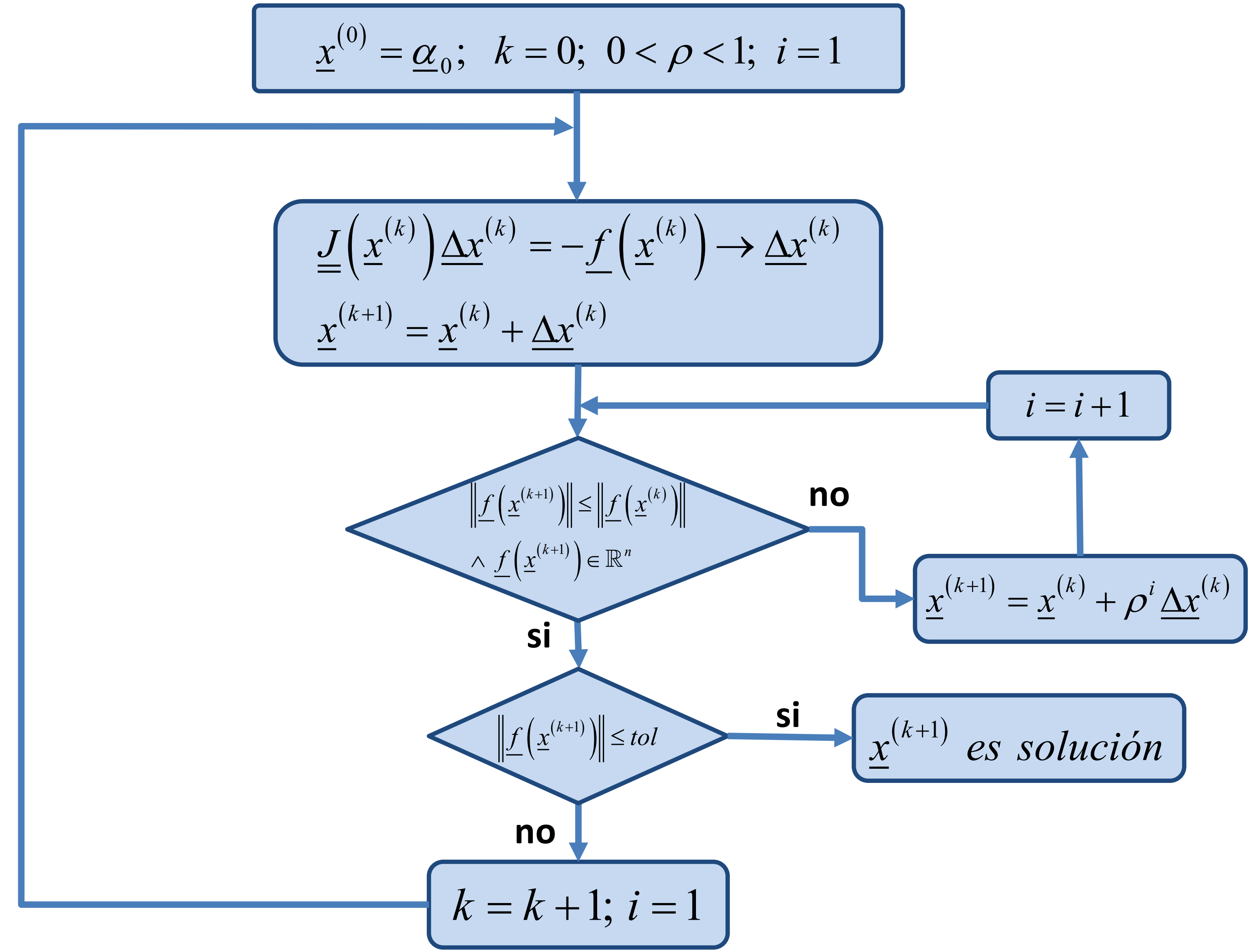
$$\underline{\underline{J}}\left(\underline{x}^{(k)}\right)\underline{\Delta x}^{(k)} = -\underline{f}\left(\underline{x}^{(k)}\right)$$

$$\underline{x}^{(k+1)} = \underline{x}^{(k)} + \rho\,\underline{\Delta x}^{(k)}$$

Factor de relajación ρ

- Se utiliza para reducir el tamaño del salto de Newton.
- Se lo reduce hasta garantizar una disminución del error. $\left\|\underline{f}\left(\underline{x}^{(k+1)}\right)\right\| \leq \left\|\underline{f}\left(\underline{x}^{(k)}\right)\right\|$
- También se reduce para evitar inconsistencias matemáticas en el nuevo valor $\underline{f}\left(\underline{x}^{(k+1)}\right) \in \mathbb{R}^n$

Método de Newton (II)



Método de Newton (II - Scilab)

```
function [solution, iter]=newtonrelax(fun, Jx, x0, rho0, tol)
   x = x0;   itemax = 100;    errora = 2*tol;    iter = 1;   rho=rho0;
   while errora > tol
      f = fun(x);  error1 = max(abs(f));
      J = Jx(x);
      dx = J\(-f);
      xnew = x + dx;
      f = fun(xnew);   error2 = max(abs(f));
      i=1;   imax=10;
      while (error2 >= error1 || ~isreal(f))
         xnew = x + (rho^i)*dx;
         f = fun(xnew);   error2 = max(abs(f));
         i = i+1;
         if i > imax
            disp(' No convergence: Change relaxation factor or initial guesses')
            solution = xnew;
         end
      end
      x = xnew;
      f = fun(x);  errora = max(abs(f))
      iter = iter + 1;
      if iter > itemax
         disp(' No convergence: Change initial guesses or maximum number of iterations ')
         solution = x;
      end
   end
   solution = x;
endfunction
```

Ejemplo 4 en SciLab

Ejemplo para repaso

$$\begin{cases} x_2 + x_1^{\,2} - x_1 - 0.75 = 0 \\ x_2 + 5x_2x_1 - x_1^{\,2} = 0 \end{cases}$$

$$\underline{F}(\underline{x}) = \begin{bmatrix} \sqrt{-x_2 + x_1 + 0.75} \\ \left(-x_2 + x_1^{\,2}\right)/\left(5x_1\right) \end{bmatrix}$$

Función de aproximación

$$\underline{\underline{J}}(\underline{x}) = \begin{bmatrix} 2x_1 - 1 & 1 \\ 5x_2 - 2x_1 & 1 + 5x_1 \end{bmatrix}$$

Matriz jacobiana