Sistemas de Ecuaciones No Lineales

Profesor: Dr. Alejandro S. M. Santa Cruz
JTP: Dr. Juan Ignacio Manassaldi
Aux. 1ra: Ing. Amalia Rueda

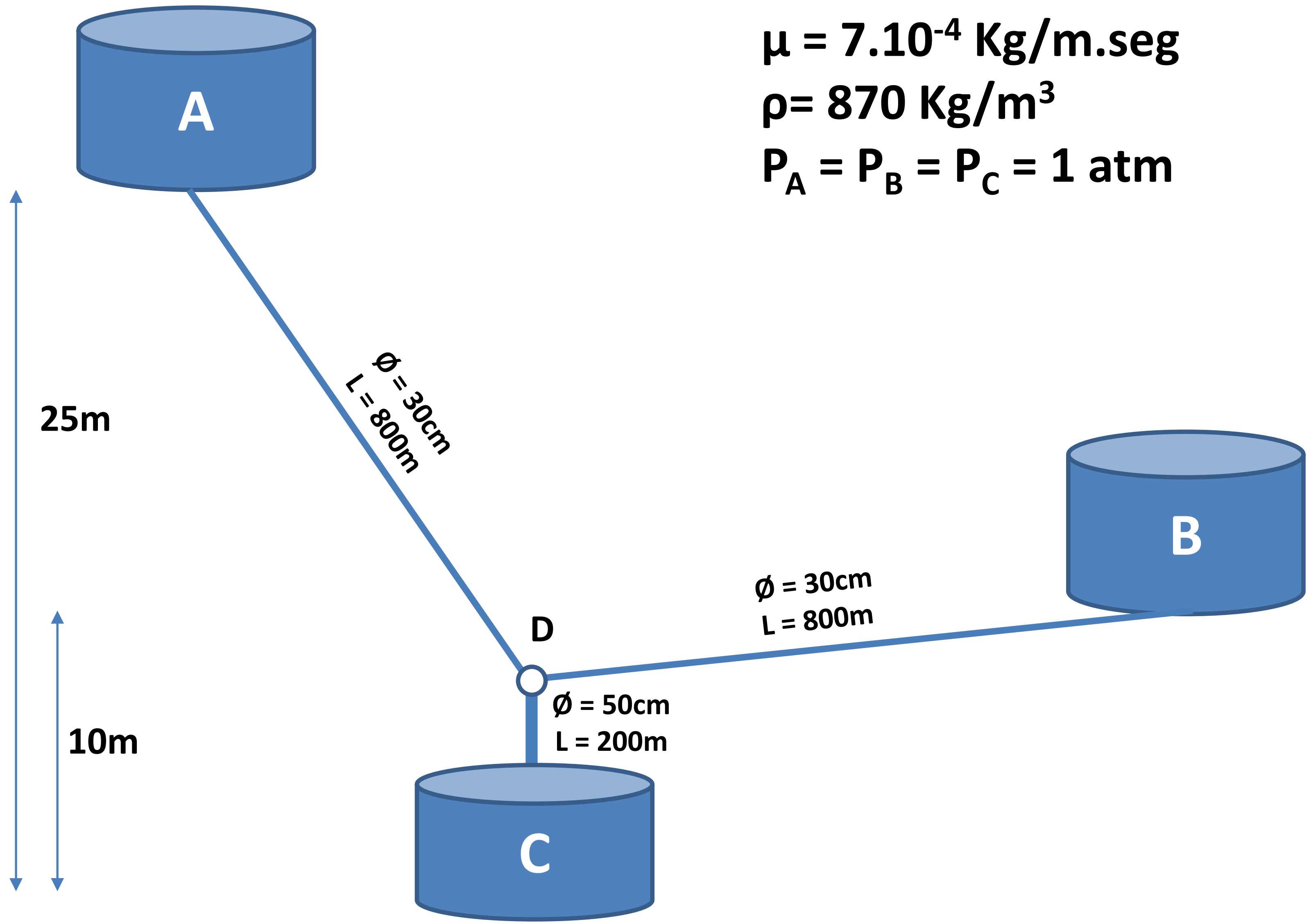
Ejemplo

Una instalación petrolífera descarga petróleo en dos depósitos A y B situados a 25m y 10m de altura sobre un tercer depósito almacén C. De los depósitos A y B parten sendas tuberías de 30cm de diámetro que confluyen en el punto D, conectándose allí con una tubería de diámetro 50cm que va hasta el deposito C. La longitud de las tuberías que parten de los depósitos A y B es de 800m y la que va desde la confluencia de la tuberías anteriores hasta C mide 200m. Si en las condiciones de transporte la viscosidad del petróleo es 7.10-4 Kg/m.seg, y la densidad 870 kg/m^3, determínese el caudal horario de petróleo descargado en C.

Extraído del libro “Problemas de Ingeniería Química” - Ocon Tojo

Capítulo 1 – Transporte de fluidos (conducciones ramificadas) Pag. 29

Ejemplo: Diagrama



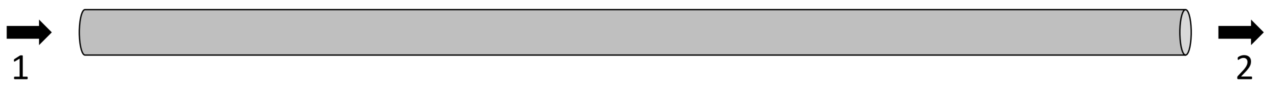
$\mu = 7.10^{-4}$ Kg/m.seg

$\rho = 870$ Kg/m^3

$P_A = P_B = P_C = 1$ atm

Ejemplo: Balance de Energía

Ecuaciones para tuberías rectas:



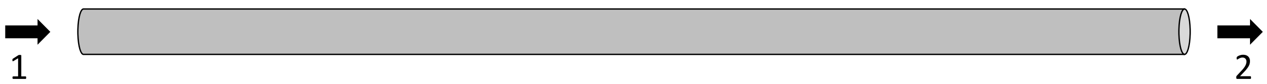
No hay trabajo ni cambios de velocidad dentro de cada cañería

$$\frac{P_2 - P_1}{g\rho} + \cancel{\frac{u_2^2 - u_1^2}{2.g}} + Z_2 - Z_1 + h_f = 0$$

Balance de Energía Mecánica

$$\frac{P_2 - P_1}{g\rho} + Z_2 - Z_1 + h_f = 0$$

Ejemplo: Balance de Energía



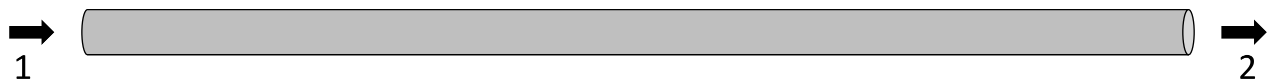
$$\frac{P_2 - P_1}{g\rho} + Z_2 - Z_1 + h_f = 0$$

Balance de Energía Mecánica

Relaciona la caída de presión, el cambio de altura y la perdida de carga por fricción de un fluido no compresible en cada tramo de cañería.

Ejemplo: Balance de Energía

Ecuaciones para tuberías rectas:



$$\frac{P_2 - P_1}{g\rho} + Z_2 - Z_1 + h_f = 0$$

$$\frac{P_2}{g\rho} - \frac{P_1}{g\rho} + Z_2 - Z_1 + h_f = 0$$

$$\left(Z_2 + \frac{P_2}{g\rho}\right) - \left(Z_1 + \frac{P_1}{g\rho}\right) + h_f = 0$$

$$h = Z + \frac{P}{g\rho}$$ Altura estática

Ejemplo: Balance de Energía

Ecuaciones para tuberías rectas:

$$\left(Z_2 + \frac{P_2}{g\rho}\right) - \left(Z_1 + \frac{P_1}{g\rho}\right) + h_f = 0$$

$$h_2 - h_1 = -h_f$$

Ejemplo: Perdida de carga por fricción

$$h_f = f\frac{L}{D}\frac{u^2}{2.g}$$

Cálculo de perdidas por fricción

Permite el cálculo de la perdida de carga por fricción h_f cuando se conoce el valor de f (factor de fricción).

Ejemplo: Factor de fricción

$$\frac{1}{\sqrt{f}} = -2Log_{10}\left(\frac{2.51}{\text{Re}.\sqrt{f}}\right)$$

Cálculo para el factor de fricción

En general, *f* se obtiene mediante gráficos (Moody) porque las buenas correlaciones son no-lineales y requieren métodos iterativos. Esta ecuación es para tubos lisos.

$$\text{Re} = \frac{u.D.\rho}{\mu}$$

Ejemplo: Resumen

$$h_2 - h_1 = -h_f$$

Balance de Energía Mecánica

$$h_f = f\,\frac{L}{D}\,\frac{u^2}{2.g}$$

Cálculo de perdidas por fricción

$$\frac{1}{\sqrt{f}} = -2Log_{10}\left(\frac{2.51}{\mathrm{Re}.\sqrt{f}}\right)$$

Cálculo para el factor de fricción

$$\mathrm{Re} = \frac{u.D.\rho}{\mu}$$

Número de Reynolds

Ejemplo: Resumen

$$h_2 - h_1 = -h_f$$

Balance de Energía Mecánica

$$h_f = f\frac{L}{D}\frac{u^2}{2.g}$$

Cálculo de perdidas por fricción

$$\frac{1}{\sqrt{f}} = -2Log_{10}\left(\frac{2.51}{\mathrm{Re}.\sqrt{f}}\right)$$

Cálculo para el factor de fricción

$$\mathrm{Re} = \frac{u.D.\rho}{\mu}$$

Número de Reynolds

Ejemplo: Resumen

$$h_2 - h_1 = -f\,\frac{L}{D}\frac{u^2}{2g}$$

$$\frac{1}{\sqrt{f}} = -2Log_{10}\left(\frac{2.51\mu}{u.D.\rho\sqrt{f}}\right)$$

Ejemplo: Resumen

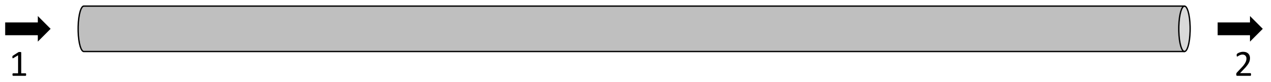
$$h_2 - h_1 = -f\,\frac{L}{D}\,\frac{u^2}{2g}$$

$$\frac{1}{\sqrt{f}} = -2Log_{10}\left(\frac{2.51\mu}{u.D.\rho\sqrt{f}}\right)$$

Redefinimos: $x = \frac{1}{\sqrt{f}} \rightarrow f = \frac{1}{x^2}$

$$h_2 - h_1 = -\frac{1}{x^2}\frac{L}{D}\frac{u^2}{2g} \qquad x = -2Log_{10}\left(\frac{2.51\mu}{u.D.\rho}x\right)$$

Ejemplo: Ecuaciones finales



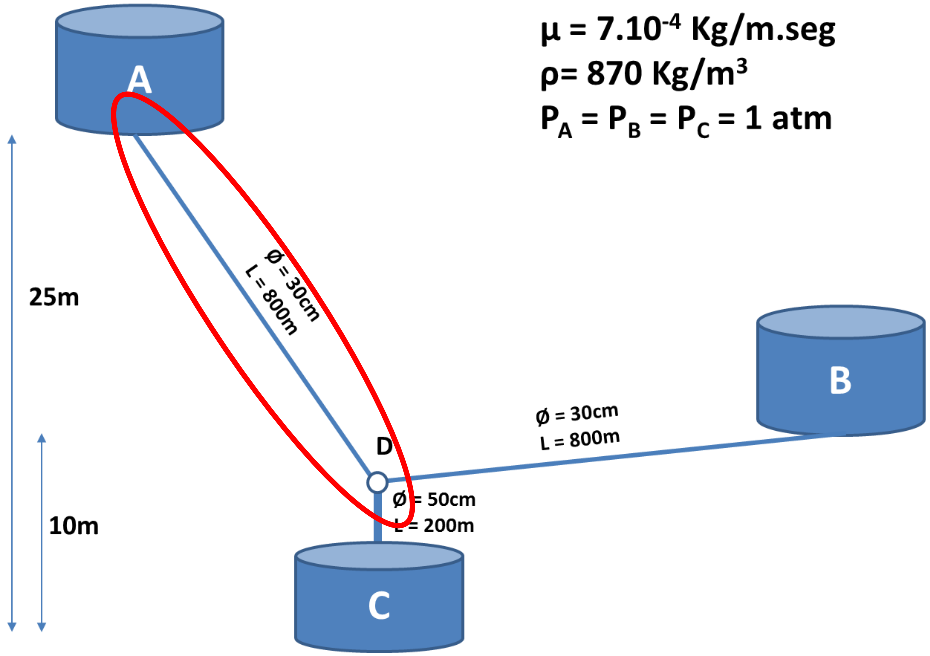
$$h_2 - h_1 = -\frac{1}{x^2}\frac{L}{D}\frac{u^2}{2g} \qquad x = -2Log_{10}\left(\frac{2.51\mu}{u.D.\rho}x\right)$$

Cada tramo debe verificar estas ecuaciones

Ejemplo: Tramo A-D

$$h_A = 25m + \frac{101325\,\frac{\mathrm{N}}{\mathrm{m}^2}}{9.8\,\frac{m}{seg^2}\,870\,\frac{kg}{m^3}}$$

$$h_A = 36.884m$$



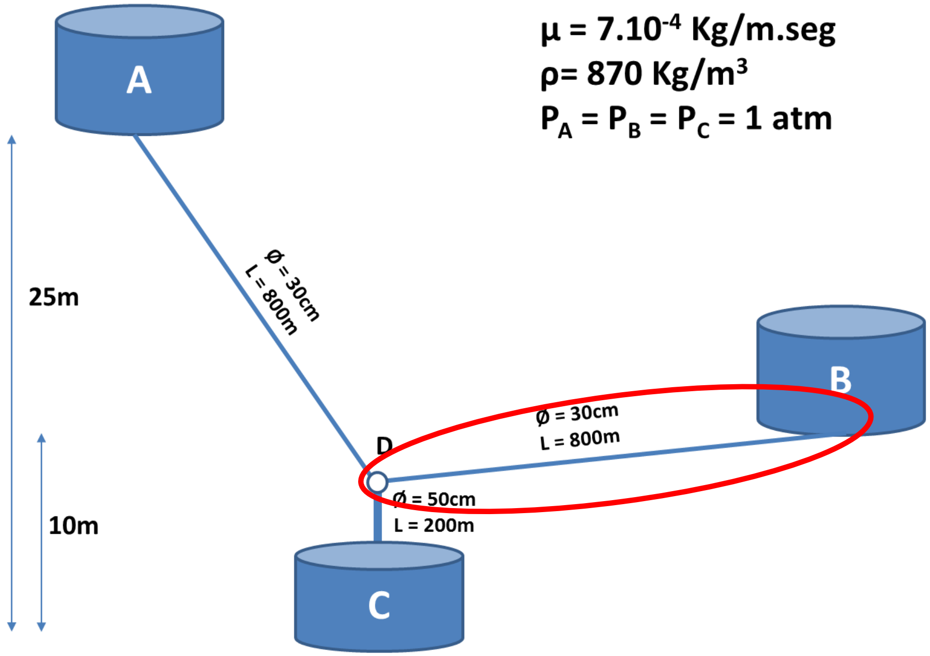
$$h_D - h_A = -\frac{1}{x_{AD}^2}\frac{L_{AD}}{D_{AD}}\frac{u_{AD}^2}{2g}$$

$$x_{AD} = -2Log_{10}\left(\frac{2.51\mu}{u_{AD}D_{AD}\rho}x_{AD}\right)$$

Ejemplo: Tramo B-D

$$h_B = 10m + \frac{101325\,\frac{\mathrm{N}}{\mathrm{m}^2}}{9.8\,\frac{m}{seg^2}\,870\,\frac{kg}{m^3}}$$

$$h_B = 21.884m$$

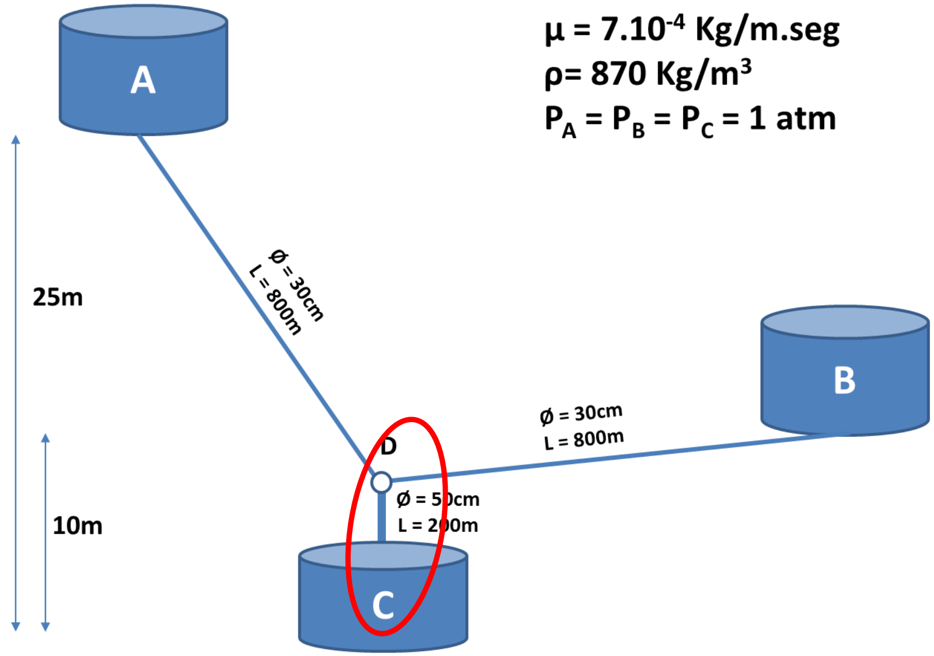


$$h_D - h_B = -\frac{1}{x_{BD}^2}\frac{L_{BD}}{D_{BD}}\frac{u_{BD}^2}{2g}$$

$$x_{BD} = -2Log_{10}\left(\frac{2.51\mu}{u_{BD}D_{BD}\rho}x_{BD}\right)$$

Ejemplo: Tramo D-C

$$h_C = 0m + \frac{101325\,\frac{\mathrm{N}}{\mathrm{m}^2}}{9.8\,\frac{m}{seg^2}\,870\,\frac{kg}{m^3}}$$



$\mu = 7.10^{-4}$ Kg/m.seg
$\rho = 870$ Kg/m^3
$P_A = P_B = P_C = 1$ atm

$$h_c = 11.884m$$

$$h_C - h_D = -\frac{1}{x_{DC}^2}\frac{L_{DC}}{D_{DC}}\frac{u_{DC}^2}{2g} \qquad x_{DC} = -2Log_{10}\left(\frac{2.51\mu}{u_{DC}D_{DC}\rho}x_{DC}\right)$$

Ejemplo: Resumen

$$h_D - h_A = -\frac{1}{x_{AD}^2}\frac{L_{AD}}{D_{AD}}\frac{u_{AD}^2}{2g} \qquad x_{AD} = -2Log_{10}\left(\frac{2.51\mu}{u_{AD}D_{AD}\rho}x_{AD}\right)$$

$$h_D - h_B = -\frac{1}{x_{BD}^2}\frac{L_{BD}}{D_{BD}}\frac{u_{BD}^2}{2g} \qquad x_{BD} = -2Log_{10}\left(\frac{2.51\mu}{u_{BD}D_{BD}\rho}x_{BD}\right)$$

$$h_C - h_D = -\frac{1}{x_{DC}^2}\frac{L_{DC}}{D_{DC}}\frac{u_{DC}^2}{2g} \qquad x_{DC} = -2Log_{10}\left(\frac{2.51\mu}{u_{DC}D_{DC}\rho}x_{DC}\right)$$

¿Incógnitas?

Ejemplo: Resumen

$$h_D - h_A = -\frac{1}{x_{AD}^2}\frac{L_{AD}}{D_{AD}}\frac{u_{AD}^2}{2g}$$

$$x_{AD} = -2Log_{10}\left(\frac{2.51\mu}{u_{AD}D_{AD}\rho}x_{AD}\right)$$

$$h_D - h_B = -\frac{1}{x_{BD}^2}\frac{L_{BD}}{D_{BD}}\frac{u_{BD}^2}{2g}$$

$$x_{BD} = -2Log_{10}\left(\frac{2.51\mu}{u_{BD}D_{BD}\rho}x_{BD}\right)$$

$$h_C - h_D = -\frac{1}{x_{DC}^2}\frac{L_{DC}}{D_{DC}}\frac{u_{DC}^2}{2g}$$

$$x_{DC} = -2Log_{10}\left(\frac{2.51\mu}{u_{DC}D_{DC}\rho}x_{DC}\right)$$

Ejemplo: Ecuaciones e Incógnitas

$$h_D - h_A = -\frac{1}{x_{AD}^2}\frac{L_{AD}}{D_{AD}}\frac{u_{AD}^2}{2g} \qquad x_{AD} = -2Log_{10}\left(\frac{2.51\mu}{u_{AD}D_{AD}\rho}x_{AD}\right)$$

$$h_D - h_B = -\frac{1}{x_{BD}^2}\frac{L_{BD}}{D_{BD}}\frac{u_{BD}^2}{2g} \qquad x_{BD} = -2Log_{10}\left(\frac{2.51\mu}{u_{BD}D_{BD}\rho}x_{BD}\right)$$

$$h_C - h_D = -\frac{1}{x_{DC}^2}\frac{L_{DC}}{D_{DC}}\frac{u_{DC}^2}{2g} \qquad x_{DC} = -2Log_{10}\left(\frac{2.51\mu}{u_{DC}D_{DC}\rho}x_{DC}\right)$$

$$h_D \quad x_{DC} \quad u_{DC} \quad x_{AD} \quad u_{AD} \quad x_{BD} \quad u_{BD}$$

Nro. de ecuaciones: 6

Nro. de incógnitas: 7

¿Podemos Resolverlo?

NO

Ejemplo: Balance de materia en D

El flujo de masa que ingresa a D es igual que el que lo abandona

$$\dot{m}_{AD} + \dot{m}_{BD} = \dot{m}_{DC}$$

$$\boxed{\dot{m} = u.A.\rho}$$

$$u_{AD}.A_{AD}.\cancel{\rho} + u_{BD}.A_{BD}.\cancel{\rho} = u_{DC}.A_{DC}.\cancel{\rho}$$

$$u_{AD}.A_{AD} + u_{BD}.A_{BD} = u_{DC}.A_{DC}$$

$$u_{AD}\cancel{\frac{\pi}{4}}D_{AD}^2 + u_{BD}\cancel{\frac{\pi}{4}}D_{BD}^2 = u_{DC}\cancel{\frac{\pi}{4}}D_{DC}^2$$

$$u_{AD}\frac{D_{AD}^2}{D_{DC}^2} + u_{BD}\frac{D_{BD}^2}{D_{DC}^2} = u_{DC}$$

Ejemplo: Ecuaciones e Incógnitas

$$h_D - h_A = -\frac{1}{x_{AD}^2}\frac{L_{AD}}{D_{AD}}\frac{u_{AD}^2}{2g} \qquad x_{AD} = -2Log_{10}\left(\frac{2.51\mu}{u_{AD}D_{AD}\rho}x_{AD}\right)$$

$$h_D - h_B = -\frac{1}{x_{BD}^2}\frac{L_{BD}}{D_{BD}}\frac{u_{BD}^2}{2g} \qquad x_{BD} = -2Log_{10}\left(\frac{2.51\mu}{u_{BD}D_{BD}\rho}x_{BD}\right)$$

$$h_C - h_D = -\frac{1}{x_{DC}^2}\frac{L_{DC}}{D_{DC}2g}\left(u_{AD}\frac{D_{AD}^2}{D_{DC}^2}+u_{BD}\frac{D_{BD}^2}{D_{DC}^2}\right)^2 \qquad x_{DC} = -2Log_{10}\left(\left(u_{AD}\frac{D_{AD}^2}{D_{DC}^2}+u_{BD}\frac{D_{BD}^2}{D_{DC}^2}\right)^{-1}\frac{2.51\mu\, x_{DC}}{D_{DC}\rho}\right)$$

$$h_D \quad x_{DC} \quad x_{AD} \quad u_{AD} \quad x_{BD} \quad u_{BD}$$

Nro. de ecuaciones: 6

Nro. de incógnitas: 6

¿Podemos Resolverlo?

SI

Sistema de ecuaciones y vector de incógnitas

$$h_D - h_A + \frac{1}{x_{AD}^2}\frac{L_{AD}}{D_{AD}}\frac{u_{AD}^2}{2g} = 0$$

$$h_D - h_B + \frac{1}{x_{BD}^2}\frac{L_{BD}}{D_{BD}}\frac{u_{BD}^2}{2g} = 0$$

$$h_C - h_D + \frac{1}{x_{DC}^2}\frac{L_{DC}}{D_{DC}2g}\left(u_{AD}\frac{D_{AD}^2}{D_{DC}^2} + u_{BD}\frac{D_{BD}^2}{D_{DC}^2}\right)^2 = 0$$

$$x_{AD} + 2Log_{10}\left(\frac{2.51\mu}{u_{AD}D_{AD}\rho}x_{AD}\right) = 0$$

$$x_{BD} + 2Log_{10}\left(\frac{2.51\mu}{u_{BD}D_{BD}\rho}x_{BD}\right) = 0$$

$$x_{DC} + 2Log_{10}\left(\left(u_{AD}\frac{D_{AD}^2}{D_{DC}^2} + u_{BD}\frac{D_{BD}^2}{D_{DC}^2}\right)^{-1}\frac{2.51\mu\, x_{DC}}{D_{DC}\rho}\right) = 0$$

Sistema de ecuaciones y vector de incógnitas

$$\underline{f}\begin{pmatrix} h_D - h_A + \dfrac{1}{x_{AD}^2}\dfrac{L_{AD}}{D_{AD}}\dfrac{u_{AD}^2}{2g} \\ h_D - h_B + \dfrac{1}{x_{BD}^2}\dfrac{L_{BD}}{D_{BD}}\dfrac{u_{BD}^2}{2g} \\ h_C - h_D + \dfrac{1}{x_{DC}^2}\dfrac{L_{DC}}{D_{DC}2g}\left(u_{AD}\dfrac{D_{AD}^2}{D_{DC}^2} + u_{BD}\dfrac{D_{BD}^2}{D_{DC}^2}\right)^2 \\ x_{AD} + 2Log_{10}\left(\dfrac{2.51\mu}{u_{AD}D_{AD}\rho}x_{AD}\right) \\ x_{BD} + 2Log_{10}\left(\dfrac{2.51\mu}{u_{BD}D_{BD}\rho}x_{BD}\right) \\ x_{DC} + 2Log_{10}\left(\left(u_{AD}\dfrac{D_{AD}^2}{D_{DC}^2} + u_{BD}\dfrac{D_{BD}^2}{D_{DC}^2}\right)^{-1}\dfrac{2.51\mu\, x_{DC}}{D_{DC}\rho}\right) \end{pmatrix} \rightarrow \underline{f}\left(\underline{x}\right) = \underline{0}$$

Sistema de ecuaciones y vector de incógnitas

$$\underline{x} = \begin{pmatrix} u_{AD} \\ u_{BD} \\ h_D \\ x_{AD} \\ x_{BD} \\ x_{DC} \end{pmatrix}$$

Aproximaciones Sucesivas

$$\underline{f}\begin{pmatrix} h_D - h_A + \dfrac{1}{x_{AD}^2}\dfrac{L_{AD}}{D_{AD}}\dfrac{u_{AD}^2}{2g} \\ h_D - h_B + \dfrac{1}{x_{BD}^2}\dfrac{L_{BD}}{D_{BD}}\dfrac{u_{BD}^2}{2g} \\ h_C - h_D + \dfrac{1}{x_{DC}^2}\dfrac{L_{DC}}{D_{DC}2g}\left(u_{AD}\dfrac{D_{AD}^2}{D_{DC}^2} + u_{BD}\dfrac{D_{BD}^2}{D_{DC}^2}\right)^2 \\ x_{AD} + 2Log_{10}\left(\dfrac{2.51\mu}{u_{AD}D_{AD}\rho}x_{AD}\right) \\ x_{BD} + 2Log_{10}\left(\dfrac{2.51\mu}{u_{BD}D_{BD}\rho}x_{BD}\right) \\ x_{DC} + 2Log_{10}\left(\left(u_{AD}\dfrac{D_{AD}^2}{D_{DC}^2} + u_{BD}\dfrac{D_{BD}^2}{D_{DC}^2}\right)^{-1}\dfrac{2.51\mu\, x_{DC}}{D_{DC}\rho}\right) \end{pmatrix} \qquad \underline{x} = \begin{pmatrix} u_{AD} \\ u_{BD} \\ h_D \\ x_{AD} \\ x_{BD} \\ x_{DC} \end{pmatrix}$$

$$
\underline{F}\begin{pmatrix}
\sqrt{\left(h_A-h_D\right)\left(x_{AD}^2 D_{AD}\, 2g\right)/L_{AD}} \\
\sqrt{\left(h_B-h_D\right)\left(x_{BD}^2 D_{BD}\, 2g\right)/L_{BD}} \\
h_C+\dfrac{1}{x_{DC}^2}\dfrac{L_{DC}}{D_{DC}\, 2g}\left(u_{AD}\dfrac{D_{AD}^2}{D_{DC}^2}+u_{BD}\dfrac{D_{BD}^2}{D_{DC}^2}\right)^2 \\
-2Log_{10}\left(\dfrac{2.51\mu}{u_{AD}D_{AD}\rho}x_{AD}\right) \\
-2Log_{10}\left(\dfrac{2.51\mu}{u_{BD}D_{BD}\rho}x_{BD}\right) \\
-2Log_{10}\left(\left(u_{AD}\dfrac{D_{AD}^2}{D_{DC}^2}+u_{BD}\dfrac{D_{BD}^2}{D_{DC}^2}\right)^{-1}\dfrac{2.51\mu\, x_{DC}}{D_{DC}\rho}\right)
\end{pmatrix}
\qquad
\underline{x}=\begin{pmatrix} u_{AD} \\ u_{BD} \\ h_D \\ x_{AD} \\ x_{BD} \\ x_{DC} \end{pmatrix}
$$

Aproximaciones Sucesivas

$$\underline{F}\begin{pmatrix} \sqrt{(h_A-h_D)\left(x_{AD}^2 D_{AD}2g\right)/L_{AD}} \\ \sqrt{(h_B-h_D)\left(x_{BD}^2 D_{BD}2g\right)/L_{BD}} \\ h_C+\frac{1}{x_{DC}^2}\frac{L_{DC}}{D_{DC}2g}\left(u_{AD}\frac{D_{AD}^2}{D_{DC}^2}+u_{BD}\frac{D_{BD}^2}{D_{DC}^2}\right)^2 \\ -2Log_{10}\left(\frac{2.51\mu}{u_{AD}D_{AD}\rho}x_{AD}\right) \\ -2Log_{10}\left(\frac{2.51\mu}{u_{BD}D_{BD}\rho}x_{BD}\right) \\ -2Log_{10}\left(\left(u_{AD}\frac{D_{AD}^2}{D_{DC}^2}+u_{BD}\frac{D_{BD}^2}{D_{DC}^2}\right)^{-1}\frac{2.51\mu\, x_{DC}}{D_{DC}\rho}\right) \end{pmatrix} \qquad \underline{x}=\begin{pmatrix} u_{AD} \\ u_{BD} \\ h_D \\ x_{AD} \\ x_{BD} \\ x_{DC} \end{pmatrix}$$

Valor de arranque

¡Es Clave!
Hay que interpretar físicamente el problema

$$h_D = 15$$

$$\mathrm{Re} = 1e6$$

$$x = -2\log_{10}\left(\frac{5.1286}{\mathrm{Re}^{0.89}}\right) \rightarrow x = 9.26$$

Ecuación de Bahr

$$\mathrm{Re} = \frac{u.D.\rho}{\mu} \rightarrow u = \frac{\mathrm{Re}\,\mu}{D.\rho} = 2.6819$$

$$\underline{x}^{(0)} = \begin{pmatrix} 2.6819 \\ 2.6819 \\ 15 \\ 9.26 \\ 9.26 \\ 9.26 \end{pmatrix}$$

Aproximaciones Sucesivas

$$\underline{x}^{(0)} = \begin{pmatrix} 2.6819 \\ 2.6819 \\ 15 \\ 9.26 \\ 9.26 \\ 9.26 \end{pmatrix} \qquad f\left(\underline{x}^{(0)}\right) = \begin{pmatrix} -10.471 \\ 4.5281 \\ -2.2283 \\ -0.0074 \\ -0.0074 \\ -0.1657 \end{pmatrix} \qquad \left\| f\left(\underline{x}^{(0)}\right) \right\| = 11.6257$$

$$\underline{x}^{(1)} = F\left(\underline{x}^{(0)}\right) = \begin{pmatrix} 3.7138 \\ 2.0829 \\ 12.7716 \\ 9.2674 \\ 9.2674 \\ 9.4257 \end{pmatrix} \qquad f\left(\underline{x}^{(1)}\right) = \begin{pmatrix} -2.263 \\ -2.2393 \\ 0.1129 \\ -0.282 \\ 0.2202 \\ -0.0520 \end{pmatrix} \qquad \left\| f\left(\underline{x}^{(1)}\right) \right\| = 3.2063$$

Función de Aproximación (I)

$$\underline{x}^{(12)} = \begin{pmatrix} 4.0124 \\ 2.3327 \\ 13.0529 \\ 9.5871 \\ 9.1560 \\ 9.5453 \end{pmatrix} \qquad f\left(\underline{x}^{(12)}\right) = \begin{pmatrix} -2.07e-5 \\ 1.42e-5 \\ 2e-7 \\ 1.4e-6 \\ -2.5e-6 \\ -6.621e-8 \end{pmatrix} \qquad \left\| f\left(\underline{x}^{(12)}\right) \right\| = 0.0000252$$

Implementación en SciLab

```
function out=sistema(x)
//x=(1: uAD,2: uBD,3: hD,4: xAD,5: xBD,6: xDC)
g=9.8  ;   //m/s2
mu=7e-4;   //kg/(m.s)
rho=870;   //kg/m3
PA=101325; //N/m2
PB=101325; //N/m2
PC=101325; //N/m2
zA=25;     //m
zB=10;     //m
zC=0;      //m
DAD=0.3;   //m
DBD=0.3;   //m
DDC=0.5;   //m
LAD=800;   //m
LBD=800;   //m
LDC=200;   //m
hA=zA + PA/(g*rho);
hB=zB + PB/(g*rho);
```

Implementación en SciLab


```
hC=zC + PC/(g*rho);
uAD=x(1);
uBD=x(2);
hD =x(3);
xAD=x(4);
xBD=x(5);
xDC=x(6);
uDC = (uAD*DAD^2 + uBD*DBD^2)/(DDC^2);

out(1,1) = hD - hA + LAD*(uAD^2)/((xAD^2)*DAD*2*g);
out(2,1) = hD - hB + LBD*(uBD^2)/((xBD^2)*DBD*2*g);
out(3,1) = hC - hD + LDC*(uDC^2)/((xDC^2)*DDC*2*g);
out(4,1) = xAD + 2*log10(2.51*mu*xAD/(uAD*DAD*rho));
out(5,1) = xBD + 2*log10(2.51*mu*xBD/(uBD*DBD*rho));
out(6,1) = xDC + 2*log10(2.51*mu*xDC/(uDC*DDC*rho));
endfunction
```

Función de aproximación en SciLab

Solo se debe modificar la salida de la función anterior

```
out(1,1) = sqrt((hA-hD)*((xAD^2)*DAD*2*g)/LAD);
out(2,1) = sqrt((hB-hD)*((xBD^2)*DBD*2*g)/LBD);
out(3,1) = hC + LDC*(uDC^2)/((xDC^2)*DDC*2*g);
out(4,1) = -2*log10(2.51*mu*xAD/(uAD*DAD*rho));
out(5,1) = -2*log10(2.51*mu*xBD/(uBD*DBD*rho));
out(6,1) = -2*log10(2.51*mu*xDC/(uDC*DDC*rho));
```

Método de Newton

$$\underline{f}\begin{pmatrix} h_D - h_A + \dfrac{1}{x_{AD}^2}\dfrac{L_{AD}}{D_{AD}}\dfrac{u_{AD}^2}{2g} \\ h_D - h_B + \dfrac{1}{x_{BD}^2}\dfrac{L_{BD}}{D_{BD}}\dfrac{u_{BD}^2}{2g} \\ h_C - h_D + \dfrac{1}{x_{DC}^2}\dfrac{L_{DC}}{D_{DC}2g}\left(u_{AD}\dfrac{D_{AD}^2}{D_{DC}^2} + u_{BD}\dfrac{D_{BD}^2}{D_{DC}^2}\right)^2 \\ x_{AD} + 2Log_{10}\left(\dfrac{2.51\mu}{u_{AD}D_{AD}\rho}x_{AD}\right) \\ x_{BD} + 2Log_{10}\left(\dfrac{2.51\mu}{u_{BD}D_{BD}\rho}x_{BD}\right) \\ x_{DC} + 2Log_{10}\left(\left(u_{AD}\dfrac{D_{AD}^2}{D_{DC}^2} + u_{BD}\dfrac{D_{BD}^2}{D_{DC}^2}\right)^{-1}\dfrac{2.51\mu\, x_{DC}}{D_{DC}\rho}\right) \end{pmatrix}$$

$$\underline{x} = \begin{pmatrix} u_{AD} \\ u_{BD} \\ h_D \\ x_{AD} \\ x_{BD} \\ x_{DC} \end{pmatrix}$$

Matriz Jacobiana

$$f_1\left(u_{AD},u_{BD},h_D,x_{AD},x_{BD},x_{DC}\right)=h_D-h_A+\frac{1}{x_{AD}^2}\frac{L_{AD}}{D_{AD}}\frac{u_{AD}^2}{2g}$$

$$f_2\left(u_{AD},u_{BD},h_D,x_{AD},x_{BD},x_{DC}\right)=h_D-h_B+\frac{1}{x_{BD}^2}\frac{L_{BD}}{D_{BD}}\frac{u_{BD}^2}{2g}$$

$$f_3\left(u_{AD},u_{BD},h_D,x_{AD},x_{BD},x_{DC}\right)=h_C-h_D+\frac{1}{x_{DC}^2}\frac{L_{DC}}{D_{DC}2g}\left(u_{AD}\frac{D_{AD}^2}{D_{DC}^2}+u_{BD}\frac{D_{BD}^2}{D_{DC}^2}\right)^2$$

$$f_4\left(u_{AD},u_{BD},h_D,x_{AD},x_{BD},x_{DC}\right)=x_{AD}+2Log_{10}\left(\frac{2.51\mu}{u_{AD}D_{AD}\rho}x_{AD}\right)$$

$$f_5\left(u_{AD},u_{BD},h_D,x_{AD},x_{BD},x_{DC}\right)=x_{BD}+2Log_{10}\left(\frac{2.51\mu}{u_{BD}D_{BD}\rho}x_{BD}\right)$$

$$f_6\left(u_{AD},u_{BD},h_D,x_{AD},x_{BD},x_{DC}\right)=x_{DC}+2Log_{10}\left(\left(u_{AD}\frac{D_{AD}^2}{D_{DC}^2}+u_{BD}\frac{D_{BD}^2}{D_{DC}^2}\right)^{-1}\frac{2.51\mu\, x_{DC}}{D_{DC}\rho}\right)$$

Matriz Jacobiana

$$f_1\left(u_{AD},u_{BD},h_D,x_{AD},x_{BD},x_{DC}\right)=h_D-h_A+\frac{1}{x_{AD}^2}\frac{L_{AD}}{D_{AD}}\frac{u_{AD}^2}{2g}$$

$$\nabla^T f_1\left(\underline{x}\right)=\begin{pmatrix}\frac{2}{x_{AD}^2}\frac{L_{AD}}{D_{AD}}\frac{u_{AD}}{2g} & 0 & 1 & \frac{-2}{x_{AD}^3}\frac{L_{AD}}{D_{AD}}\frac{u_{AD}^2}{2g} & 0 & 0\end{pmatrix}$$

$$f_2\left(u_{AD},u_{BD},h_D,x_{AD},x_{BD},x_{DC}\right)=h_D-h_B+\frac{1}{x_{BD}^2}\frac{L_{BD}}{D_{BD}}\frac{u_{BD}^2}{2g}$$

$$\nabla^T f_2\left(\underline{x}\right)=\begin{pmatrix}0 & \frac{2}{x_{BD}^2}\frac{L_{BD}}{D_{BD}}\frac{u_{BD}}{2g} & 1 & 0 & \frac{-2}{x_{BD}^3}\frac{L_{BD}}{D_{BD}}\frac{u_{BD}^2}{2g} & 0\end{pmatrix}$$

$$f_3\left(u_{AD},u_{BD},h_D,x_{AD},x_{BD},x_{DC}\right)=h_C-h_D+\frac{1}{x_{DC}^2}\frac{L_{DC}}{D_{DC}2g}\left(u_{AD}\frac{D_{AD}^2}{D_{DC}^2}+u_{BD}\frac{D_{BD}^2}{D_{DC}^2}\right)^2$$

$$\nabla^T f_3\left(\underline{x}\right)=\left(\frac{2}{x_{DC}^2}\frac{L_{DC}}{D_{DC}2g}\left(u_{AD}\frac{D_{AD}^2}{D_{DC}^2}+u_{BD}\frac{D_{BD}^2}{D_{DC}^2}\right)\frac{D_{AD}^2}{D_{DC}^2}\quad \frac{2}{x_{DC}^2}\frac{L_{DC}}{D_{DC}2g}\left(u_{AD}\frac{D_{AD}^2}{D_{DC}^2}+u_{BD}\frac{D_{BD}^2}{D_{DC}^2}\right)\frac{D_{BD}^2}{D_{DC}^2}\right.$$

$$\left.\ldots -1\quad 0\quad 0\quad \frac{-2}{x_{DC}^3}\frac{L_{DC}}{D_{DC}2g}\left(u_{AD}\frac{D_{AD}^2}{D_{DC}^2}+u_{BD}\frac{D_{BD}^2}{D_{DC}^2}\right)^2\right)$$

$$f_4\left(u_{AD},u_{BD},h_D,x_{AD},x_{BD},x_{DC}\right)=x_{AD}+2Log_{10}\left(\frac{2.51\mu}{u_{AD}D_{AD}\rho}x_{AD}\right)$$

$$\nabla^T f_4\left(\underline{x}\right)=\left(\frac{-2log_{10}\left(e\right)}{u_{AD}}\quad 0\quad 0\quad 1+\frac{2log_{10}\left(e\right)}{x_{AD}}\quad 0\quad 0\right)$$

Matriz Jacobiana

$$f_5\left(u_{AD},u_{BD},h_D,x_{AD},x_{BD},x_{DC}\right)=x_{BD}+2Log_{10}\left(\frac{2.51\mu}{u_{BD}D_{BD}\rho}x_{BD}\right)$$

$$\nabla^T f_5\left(\underline{x}\right)=\begin{pmatrix}0 & \dfrac{-2log_{10}\left(e\right)}{u_{BD}} & 0 & 0 & 1+\dfrac{2log_{10}\left(e\right)}{x_{BD}} & 0\end{pmatrix}$$

$$f_6\left(u_{AD},u_{BD},h_D,x_{AD},x_{BD},x_{DC}\right)=x_{DC}+2Log_{10}\left(\left(u_{AD}\frac{D_{AD}^2}{D_{DC}^2}+u_{BD}\frac{D_{BD}^2}{D_{DC}^2}\right)^{-1}\frac{2.51\mu\, x_{DC}}{D_{DC}\rho}\right)$$

$$=x_{DC}-2Log_{10}\left(u_{AD}\frac{D_{AD}^2}{D_{DC}^2}+u_{BD}\frac{D_{BD}^2}{D_{DC}^2}\right)+2Log_{10}\left(\frac{2.51\mu\, x_{DC}}{D_{DC}\rho}\right)$$

$$\nabla^T f_6\left(\underline{x}\right)=\begin{pmatrix}\dfrac{-2log_{10}\left(e\right)\dfrac{D_{AD}^2}{D_{DC}^2}}{\left(u_{AD}\dfrac{D_{AD}^2}{D_{DC}^2}+u_{BD}\dfrac{D_{BD}^2}{D_{DC}^2}\right)} & \dfrac{-2log_{10}\left(e\right)\dfrac{D_{BD}^2}{D_{DC}^2}}{\left(u_{AD}\dfrac{D_{AD}^2}{D_{DC}^2}+u_{BD}\dfrac{D_{BD}^2}{D_{DC}^2}\right)} & 0 & 0 & 0 & 1+\dfrac{2log_{10}\left(e\right)}{x_{DC}}\end{pmatrix}$$

$$\underline{x}^{(0)} = \begin{pmatrix} 2.6819 \\ 2.6819 \\ 15 \\ 9.26 \\ 9.26 \\ 9.26 \end{pmatrix} \qquad f\left(\underline{x}^{(0)}\right) = \begin{pmatrix} -10.471 \\ 4.5281 \\ -2.2283 \\ -0.0074 \\ -0.0074 \\ -0.1657 \end{pmatrix} \qquad \left\| f\left(\underline{x}^{(0)}\right) \right\| = 11.6257$$

$$\underline{x}^{(1)} = \underline{x}^{(0)} - J^{-1}\left(\underline{x}^{(0)}\right) f\left(\underline{x}^{(0)}\right) = \begin{pmatrix} 4.27138 \\ 2.34356 \\ 13.12111 \\ 9.73740 \\ 9.16658 \\ 9.59678 \end{pmatrix} \quad f\left(\underline{x}^{(1)}\right) = \begin{pmatrix} 2.41641 \\ 0.12989 \\ 0.19765 \\ 0.10941 \\ 0.00751 \\ 0.01994 \end{pmatrix} \quad \left\| f\left(\underline{x}^{(1)}\right) \right\| = 2.42255$$

$$\underline{x}^{(2)} = \underline{x}^{(1)} - J^{-1}\left(\underline{x}^{(1)}\right) f\left(\underline{x}^{(1)}\right) = \begin{pmatrix} 4.01476 \\ 2.33279 \\ 13.0525 \\ 9.58904 \\ 9.15608 \\ 9.54630 \end{pmatrix}$$

$$\left\| f\left(\underline{x}^{(2)}\right) \right\| = 0.01806$$

$$\underline{x}^{(3)} = \begin{pmatrix} 4.0124338 \\ 2.3327250 \\ 13.052958 \\ 9.5871723 \\ 9.1560484 \\ 9.5453420 \end{pmatrix}$$

$$\left\| f\left(\underline{x}^{(3)}\right) \right\| = 1.478449 \times 10^{-7}$$

Matriz Jacobiana en SciLab

Solo se debe modificar la salida de la función anterior

```
out(1,1) = 2*LAD*uAD/((xAD^2)*DAD*2*g);
out(1,2) = 0;
out(1,3) = 1;
out(1,4) = -2*LAD*(uAD^2)/((xAD^3)*DAD*2*g);
out(1,5) = 0;
out(1,6) = 0;
out(2,1) = 0;
out(2,2) = 2*LBD*uBD/((xBD^2)*DBD*2*g);
out(2,3) = 1;
out(2,4) = 0;
out(2,5) = -2*LBD*(uBD^2)/((xBD^3)*DBD*2*g);
out(2,6) = 0;
```

Matriz Jacobiana (en SciLab)

```
out(3,1) = 2*LDC*(uDC)*(DAD^2/DDC^2)/((xDC^2)*DDC*2*g);
out(3,2) = 2*LDC*(uDC)*(DBD^2/DDC^2)/((xDC^2)*DDC*2*g);
out(3,3) = -1;
out(3,4) = 0;
out(3,5) = 0;
out(3,6) = -2*LDC*(uDC^2)/((xDC^3)*DDC*2*g);
out(4,1) = -2*log10(%e)/uAD;
out(4,2) = 0;
out(4,3) = 0;
out(4,4) = 1 + 2*log10(%e)/xAD;
out(4,5) = 0;
out(4,6) = 0;
```

Matriz Jacobiana (en SciLab)

```
out(5,1) = 0;
out(5,2) = -2*log10(%e)/uBD;
out(5,3) = 0;
out(5,4) = 0;
out(5,5) = 1 + 2*log10(%e)/xBD;
out(5,6) = 0;
out(6,1) = -2*log10(%e)*(DAD^2/DDC^2)/uDC;
out(6,2) = -2*log10(%e)*(DBD^2/DDC^2)/uDC;
out(6,3) = 0;
out(6,4) = 0;
out(6,5) = 0;
out(6,6) = 1 + 2*log10(%e)/xDC;
```