Propagación de Errores

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Planteo del problema

A partir de una medición de temperatura deseamos conocer la presión de vapor utilizando la ecuación ampliada de Antoine

$$P_v = 10^{-2} e^{\left(65.9278 - \frac{7227.53}{T} - 7.17695 \ln(T) + 4.0313 \times 10^{-6} T^2\right)}$$

$$T = 323.15 \pm 1\ ^{o}K$$

$$P_v = 0.1235 \pm ?\ \ Bar$$

Propósito: Estudiar como los errores en los números pueden propagarse a través de las funciones matemáticas.

$f(x)$ → Función solamente dependiente de *x* $P_v = f(T)$

$\tilde{x}$ → Aproximación del valor real de *x* $\tilde{T} = 323.15\ ^{o}K$

$f(\tilde{x})$ → Estimación de ***f(x)*** $P_v = f(\tilde{T}) = 0.1235\ Bar$

$\Delta\tilde{x} = |x - \tilde{x}|$ → Error en la variable

$$\Delta\tilde{T} = |T - \tilde{T}| = 1^{o}K$$

Máximo

¿Cómo será la variación en la función?

$$\Delta f(\tilde{x}) = \left| f(x) - f(\tilde{x}) \right| \longrightarrow \Delta f(\tilde{T}) = \left| f(T) - f(\tilde{T}) \right|$$

No podemos saber el valor real de la presión de vapor porque no conocemos el valor real de la temperatura. ¿Cómo podemos estimarlo?

Pista:

- Tenemos la función y un valor cercano al real

$f(T) \quad \tilde{T}$

$$f(T) = \sum_{n=0}^{\infty} \frac{f^{(n)}(\tilde{T})}{n!} (T - \tilde{T})^n$$

$$\Delta f\left(\tilde{T}\right)=\left|f(T)-f\left(\tilde{T}\right)\right|$$

$$f(T)=f(\tilde{T})+f'(\tilde{T})(T-\tilde{T})+\frac{f''(\tilde{T})}{2!}(T-\tilde{T})^2+\frac{f'''(\tilde{T})}{3!}(T-\tilde{T})^3+\frac{f''''(\tilde{T})}{4!}(T-\tilde{T})^4+\ldots$$

$$f(T)-f(\tilde{T})=f'(\tilde{T})(T-\tilde{T})+\frac{f''(\tilde{T})}{2!}(T-\tilde{T})^2+\frac{f'''(\tilde{T})}{3!}(T-\tilde{T})^3+\frac{f''''(\tilde{T})}{4!}(T-\tilde{T})^4+\ldots$$

$$\Delta f\left(\tilde{T}\right)=\left|f(T)-f(\tilde{T})\right|=\left|f'(\tilde{T})(T-\tilde{T})+\cancel{\frac{f''(\tilde{T})}{2!}(T-\tilde{T})^2+\frac{f'''(\tilde{T})}{3!}(T-\tilde{T})^3+\frac{f''''(\tilde{T})}{4!}(T-\tilde{T})^4+\ldots}\right|$$

$$\Delta f\left(\tilde{T}\right)\cong\left|f'(\tilde{T})(T-\tilde{T})\right|$$

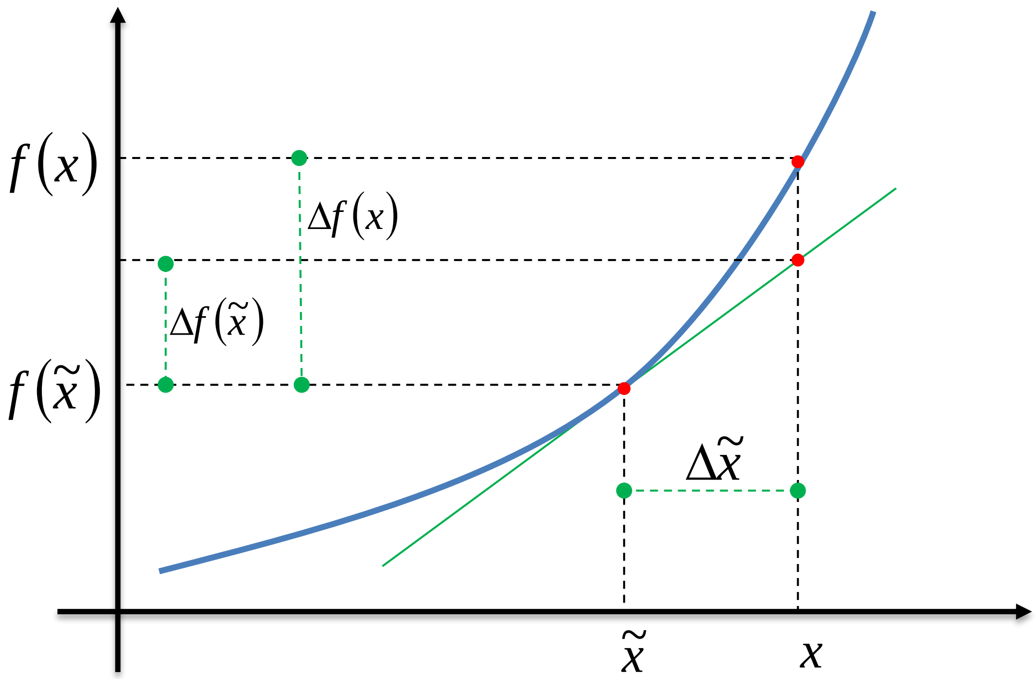
$$\Delta f\left(\tilde{T}\right)\cong\left|f'(\tilde{T})\right|\underbrace{\left|(T-\tilde{T})\right|}$$

No lo conocemos,
utilizamos su valor máximo

$\Longrightarrow$

$$\Delta f\left(\tilde{T}\right)=\left|f'(\tilde{T})\right|\Delta\tilde{T}$$

$$\Delta f\left(\tilde{x}\right)=\left|f'(\tilde{x})\right|\Delta\tilde{x}$$



$$\Delta f\left(\tilde{T}\right)=\left|f\left(T\right)-f\left(\tilde{T}\right)\right| \longrightarrow f\left(T\right)=f\left(\tilde{T}\right)\pm\Delta f\left(\tilde{T}\right)$$

$$f\left(T\right)=10^{-2}e^{\left(65.9278-\frac{7227.53}{T}-7.17695\ln\left(T\right)+4.0313\times10^{-6}T^{2}\right)}$$

$$f'\left(T\right)=f\left(T\right)\left(\frac{7227.53}{T^{2}}-\frac{7.17695}{T}+8.0626\times10^{-6}T\right)$$

1 ºK

$$\Delta f\left(\tilde{T}\right)=\left|f'(\tilde{T})\right|\Delta\tilde{T}=0.006126*1$$

325.15 ºK

$$T=323.15\ \pm1\ ^{o}K \qquad P_{v}=0.1235\ \pm0.006126\ \ Bar$$

$$T = 323.15 \ \pm 1\,^{\mathrm{o}}K$$

$$\varepsilon_T\,\% = \frac{1}{323.15}\times 100 = 0.31\%$$

$$P_v = 0.1235 \ \pm 0.006126 \ \ Bar$$

$$\varepsilon_{P_v}\,\% = \frac{0.006126}{0.1235}\times 100 = 4.96\%$$

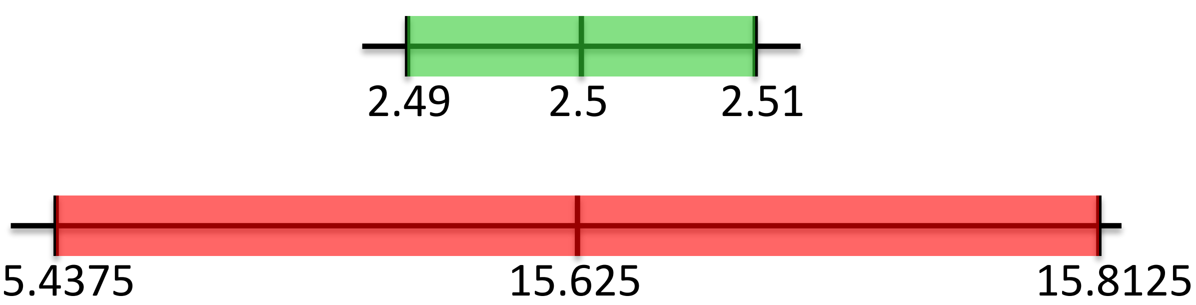
Ejemplo

Dado un valor de $\tilde{x} = 2.5$ con un error $\Delta\tilde{x} = 0.01$, estimar el error resultante en la función $f(x) = x^3$

$$\Delta f(\tilde{x}) = 3 * 2.5^2 * 0.01 = 0.1875$$

$$f(\tilde{x}) = 2.5^3 = 15.625$$

$$f(2.5) = 15.625 \pm 0.1875$$



Resumen

Realizamos una medición con incertidumbre

$$x = \tilde{x} \pm \Delta\tilde{x}$$

Estimamos el valor de la función y la propagación del error

$$f(x) = f(\tilde{x}) \pm \Delta f(\tilde{x})$$

$$\Delta f(\tilde{x}) \cong \left| f'(\tilde{x}) \right| \Delta\tilde{x}$$

Funciones de mas de una variable

Cada medición tiene una cierta con incertidumbre

$$x_1 = \tilde{x}_1 \pm \Delta\tilde{x}_1$$

$$x_2 = \tilde{x}_2 \pm \Delta\tilde{x}_2$$

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$$x_n = \tilde{x}_n \pm \Delta\tilde{x}_n$$

Estimamos el valor de la función y la propagación del error

$$f(x_1, x_2, \mathrm{K}\ , x_n) = f(\tilde{x}_1, \tilde{x}_2, \mathrm{K}\ , \tilde{x}_n) \pm \boxed{\Delta f(\tilde{x}_1, \tilde{x}_2, \mathrm{K}\ , \tilde{x}_n)}$$

$$\boxed{\Delta f(\tilde{x}_1, \tilde{x}_2, \mathrm{K}\ , \tilde{x}_n) \cong \left|\frac{\partial f}{\partial x_1}\right| \Delta\tilde{x}_1 + \left|\frac{\partial f}{\partial x_2}\right| \Delta\tilde{x}_2 + \ldots + \left|\frac{\partial f}{\partial x_n}\right| \Delta\tilde{x}_n}$$