Modelado dinámico.

Reactor Tanque Agitado con nivel controlado.

Sea el reactor de la figura. Se desea plantear el modelo dinámico que lo represente. El nivel esté controlado por la salida (acción directa).

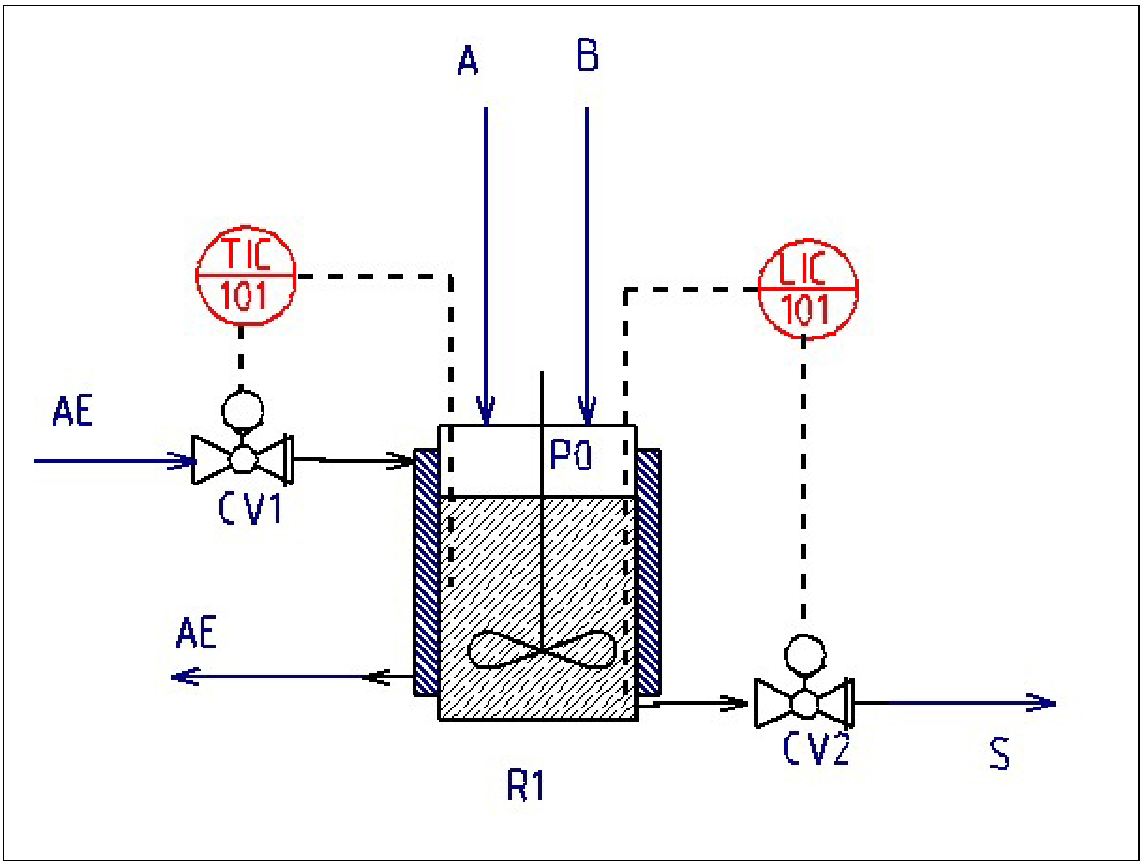

Figura 1

$$A + B \leftrightarrow C$$

Hipótesis:

- Las presiones P0 y Ps son conocidas y constantes.
- Reacción química elemental. Exotérmica
- Evolución isotérmica
- Densidad constante
- Controladores PID

Unidades:

- A y B en flujo másico
- HT, altura del tanque
- AT, área del tanque
- Conductividades de las válvulas: CV1 y CV2

- K_{P1}, K_{I1} y K_{D1}, constantes proporcionales, integrales y diferenciales del controlador de Temperatura y K_{P2}, K_{I2} y K_{D2} para el de nivel.
- S expresado en flujo másico
- Donde α_1 es una característica de la válvula CV1 y α_1 de la CV2
- h_{SP} es el set point de la altura, esto es el valor deseado de las misma mientras que Tsp es el valor deseado de temperatura
- Donde ε_T es un criterio de convergencia

$$-r_A = K_D C_A * C_B - K_I * C_C$$

Sistema de ecuaciones:

Balance de materia global:

$$\frac{dM}{dt} = A + B - S$$

$$\rho_S \frac{dV}{dt} = A + B - S$$

$$\rho_S \ A_T \frac{dh}{dt} = A + B - S$$

$$\frac{dh}{dt} = \frac{(A + B - S)}{\rho_S \ A_T} \qquad (1)$$

Balance de materia por componente:

$$\frac{d(C_A V)}{dt} = \frac{A}{M_A} - \frac{S}{M_S} - (K_D * C_A * C_C - K_I * C_D)V$$

$$\frac{d(C_B V)}{dt} = \frac{B}{M_B} - \frac{S}{M_S} - (K_D * C_A * C_C - K_I * C_D)V$$

$$\frac{d(C_C V)}{dt} = -\frac{S}{M_S} + (K_D * C_A * C_C - K_I * C_D)\,V$$

$$\rho_S \ A_T \frac{d(C_A h)}{dt} = \frac{A}{M_A} - \frac{S}{M_S} - (K_D * C_A * C_C - K_I * C_D)V$$

$$\rho_S \ A_T \frac{d(C_B h)}{dt} = \frac{B}{M_B} - \frac{S}{M_S} - (K_D * C_A * C_C - K_I * C_D)V$$

$$\rho_S \ A_T \frac{d(C_C h)}{dt} = -\frac{S}{M_S} + (K_D * C_A * C_C - K_I * C_D)\,V$$

$$\frac{d(C_A h)}{dt}=\left[\frac{A}{M_A}-\frac{S}{M_S}-\left(K_D*C_A*C_C-K_I*C_D\right)V\right]\Big/\rho_S\, A_T$$
$$\frac{d(C_B h)}{dt}=\left[\frac{B}{M_B}-\frac{S}{M_S}-\left(K_D*C_A*C_C-K_I*C_D\right)V\right]\Big/\rho_S\, A_T$$
$$\frac{d(C_C h)}{dt}=\left[-\frac{S}{M_S}+\left(K_D*C_A*C_C-K_I*C_D\right)V\right]\Big/\rho_S\, A_T$$

$$\frac{d(C_A h)}{dt}=C_A\frac{dh}{dt}+h\frac{dC_A}{dt}=\left[\frac{A}{M_A}-\frac{S}{M_S}-\left(K_D*C_A*C_C-K_I*C_D\right)V\right]\Big/\rho_S\, A_T$$
$$\frac{d(C_B h)}{dt}=C_B\frac{dh}{dt}+h\frac{dC_B}{dt}=\left[\frac{B}{M_B}-\frac{S}{M_S}-\left(K_D*C_A*C_C-K_I*C_D\right)V\right]\Big/\rho_S\, A_T$$
$$\frac{d(C_C h)}{dt}=C_C\frac{dh}{dt}+h\frac{dC_C}{dt}=\left[-\frac{S}{M_S}+\left(K_D*C_A*C_C-K_I*C_D\right)V\right]\Big/\rho_S\, A_T$$

$$\frac{dC_A}{dt}=\left\{\left[\frac{A}{M_A}-\frac{S}{M_S}-\left(K_D*C_A*C_C-K_I*C_D\right)A_T\, h\right]\Big/\rho_S\, A_T-C_A\frac{dh}{dt}\right\}\Big/h$$
$$\frac{dC_B}{dt}=\left\{\left[\frac{B}{M_B}-\frac{S}{M_S}-\left(K_D*C_A*C_C-K_I*C_D\right)A_T\, h\right]\Big/\rho_S\, A_T-C_B\frac{dh}{dt}\right\}\Big/h \quad (2)$$
$$\frac{dC_C}{dt}=\left\{\left[-\frac{S}{M_S}+\left(K_D*C_A*C_C-K_I*C_D\right)A_T\, h\right]\Big/\rho_S\, A_T-C_C\frac{dh}{dt}\right\}\Big/h$$

Balance de energía:

$$\frac{d(\rho_S\, A_T\, H_s\, h)}{dt}=\frac{A\,H_A}{M_A}+\frac{B\,H_B}{M_B}-\frac{S\,H_S}{M_S}+(-\Delta H_R)(k_D\, C_A\, C_B-k_I\, C_C)V-Q_I$$

$$\rho_S\, A_T\frac{d(H_s\, h)}{dt}=\frac{A\,H_A}{M_A}+\frac{B\,H_B}{M_B}-\frac{S\,H_S}{M_S}+(-\Delta H_R)(k_D\, C_A\, C_B-k_I\, C_C)V-Q_I$$

$$\frac{d(H_s\, h)}{dt}=\left[\frac{A\,H_A}{M_A}+\frac{B\,H_B}{M_B}-\frac{S\,H_S}{M_S}+(-\Delta H_R)(k_D\, C_A\, C_B-k_I\, C_C)V-Q_I\right]\Big/\rho_S\, A_T$$

$$\frac{d(H_s\, h)}{dt}=H_S\frac{dh}{dt}+h\frac{dH_s}{dt}=\left[\frac{A\,H_A}{M_A}+\frac{B\,H_B}{M_B}-\frac{S\,H_S}{M_S}+(-\Delta H_R)(k_D\, C_A\, C_B-k_I\, C_C)V-Q_I\right]\Big/\rho_S\, A_T$$

$$\frac{dH_s}{dt}=\left\{\left[\frac{A\,H_A}{M_A}+\frac{B\,H_B}{M_B}-\frac{S\,H_S}{M_S}+(-\Delta H_R)(k_D\, C_A\, C_B-k_I\, C_C)V-Q_I\right]\Big/\rho_S\, A_T-H_S\frac{dh}{dt}\right\}\Big/h \quad (3)$$

Camisa de refrigeración:

$$Q_I = AE\,Cp\left(T_S - T_{AE}\right)$$

$$T_{AS} = \frac{Q_I}{AE\,Cp} + T_{AE}$$

Calor intercambiado:

$$Q_I = UA\left(T_S - T_{AS}\right)$$

Válvulas:

$$S = \rho_S\, Cv_2\, \alpha_2^{\,AC_2} \sqrt{\frac{P_F - P_S}{\rho_S}}$$

$$AE = \rho_{AE}\, Cv_1\, \alpha_1^{\,AC_1} \sqrt{\frac{P_{AE} - P_{AS}}{\rho_{AE}}}$$

$$\varepsilon_h = \left(h(t) - h_{SP}\right)$$

$$\varepsilon_T = \left(T_S - T_{SP}\right)$$

$$P_F = P_0 + \rho\, g\, h(t)$$

$$AC_1 = A_{P1} + A_{I1} + A_{D1}$$

$$A_{P1} = K_{P1}\varepsilon_T = K_{P1}\left(T_S - h_{SP}\right)$$

$$A_{I1} = K_{I1}\int_0^t \varepsilon_T\, dt \qquad \frac{dA_{I1}}{dt} = K_{I1}\,\varepsilon_T = K_{I1}\left(T_S - h_{SP}\right)$$

$$\frac{dA_{I1}}{dt} = K_{I1}\left(T_S - h_{SP}\right) \qquad (4)$$

$$A_{D1} = K_{D1}\frac{d\varepsilon_T}{dt} = K_{D1}\frac{dT_S}{dt}$$

$$AC_2 = A_{P2} + A_{I2} + A_{D2}$$

$$A_{P2} = K_{P2}\varepsilon_h = K_{P2}\left(h - h_{SP}\right)$$

$$A_{I2} = K_{I2}\int_0^t \varepsilon_h\, dt \qquad \frac{dA_{I2}}{dt} = K_{I2}\,\varepsilon_h = K_{I2}\left(h - h_{SP}\right)$$

$$\frac{dA_{I2}}{dt} = K_{I2}\left(h - h_{SP}\right) \quad (5)$$

$$A_{D2} = K_{D2}\frac{d\varepsilon_h}{dt} = K_{D2}\frac{dh}{dt}$$

Con lo que queda un sistema de ecuaciones diferenciales:

$$\frac{dh}{dt} = \frac{\left(A + B - S\right)}{\rho_S\, A_T}$$

$$\frac{dC_A}{dt} = \left\{\left[\frac{A}{M_A} - \frac{S}{M_S} - \left(K_D * C_A * C_C - K_I * C_D\right)A_T\, h\right]\Big/ \rho_S\, A_T - C_A\frac{dh}{dt}\right\}\Big/ h$$

$$\frac{dC_B}{dt} = \left\{\left[\frac{B}{M_B} - \frac{S}{M_S} - \left(K_D * C_A * C_C - K_I * C_D\right)A_T\, h\right]\Big/ \rho_S\, A_T - C_B\frac{dh}{dt}\right\}\Big/ h$$

$$\frac{dC_C}{dt} = \left\{\left[-\frac{S}{M_S} + \left(K_D * C_A * C_C - K_I * C_D\right)A_T\, h\right]\Big/ \rho_S\, A_T - C_C\frac{dh}{dt}\right\}\Big/ h$$

$$\frac{dH_s}{dt} = \left\{\left[\frac{A\,H_A}{M_A} + \frac{B\,H_B}{M_B} - \frac{S\,H_S}{M_S} + \left(-\Delta H_R\right)\left(k_D\, C_A\, C_B - k_I\, C_C\right)V - Q_I\right]\Big/ \rho_S\, A_T - H_S\frac{dh}{dt}\right\}\Big/ h$$

$$\frac{dA_{I1}}{dt} = K_{I1}\left(T_S - h_{SP}\right)$$

$$\frac{dA_{I2}}{dt} = K_{I2}\left(h - h_{SP}\right)$$