

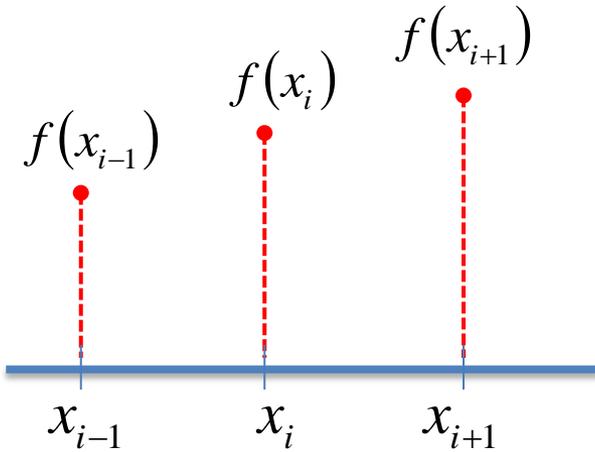
Aproximación de derivadas con valores discretos

Prof.: Dr. Alejandro S. M. Santa Cruz

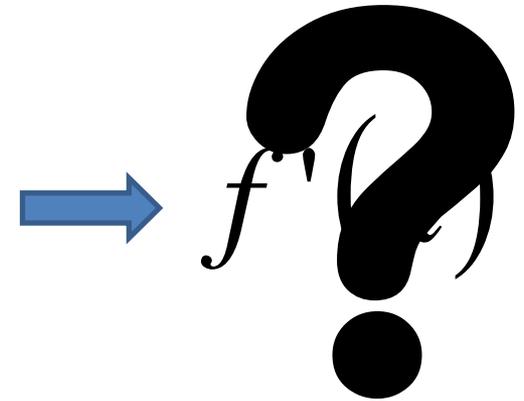
J.T.P.: Dr. Juan Ignacio Manassaldi

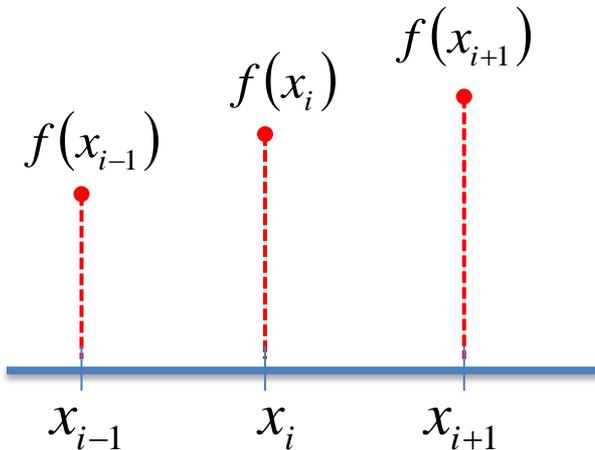
Aux. 2^{da}: Sra. Amalia Rueda

Aux. 2^{da}: Sr. Alejandro Jesús Ladreyt



x	$f(x)$
x_0	$f(x_0)$
x_1	$f(x_1)$
x_2	$f(x_2)$
x_3	$f(x_3)$
\vdots	\vdots
x_{i-1}	$f(x_{i-1})$
x_i	$f(x_i)$
x_{i+1}	$f(x_{i+1})$
\vdots	\vdots
x_n	$f(x_n)$





Serie de Taylor en torno de x_i

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(x_i)}{n!} (x - x_i)^n$$

Podemos conocer el valor en el punto siguiente

$$f(x_{i+1}) = \sum_{n=0}^{\infty} \frac{f^{(n)}(x_i)}{n!} (x_{i+1} - x_i)^n$$

$$f(x_{i+1}) = \sum_{n=0}^{\infty} \frac{f^{(n)}(x_i)}{n!} (x_{i+1} - x_i)^n$$

$$f(x_{i+1}) = f(x_i) + f'(x_i)(x_{i+1} - x_i) + \frac{f''(x_i)}{2!}(x_{i+1} - x_i)^2 + \frac{f'''(x_i)}{3!}(x_{i+1} - x_i)^3 + \frac{f^{(4)}(x_i)}{4!}(x_{i+1} - x_i)^4 + \frac{f^{(5)}(x_i)}{5!}(x_{i+1} - x_i)^5 + \dots$$

$$f'(x_i) = \frac{f(x_{i+1}) - f(x_i)}{(x_{i+1} - x_i)} - \left(\frac{f''(x_i)}{2!}(x_{i+1} - x_i) + \frac{f'''(x_i)}{3!}(x_{i+1} - x_i)^2 + \frac{f^{(4)}(x_i)}{4!}(x_{i+1} - x_i)^3 + \frac{f^{(5)}(x_i)}{5!}(x_{i+1} - x_i)^4 + \dots \right)$$

$$f'(x_i) \cong \frac{f(x_{i+1}) - f(x_i)}{(x_{i+1} - x_i)}$$

$$f(x_{i-1}) = \sum_{n=0}^{\infty} \frac{f^{(n)}(x_i)}{n!} (x_{i-1} - x_i)^n$$

$$f(x_{i-1}) = f(x_i) + f'(x_i)(x_{i-1} - x_i) + \frac{f''(x_i)}{2!}(x_{i-1} - x_i)^2 + \frac{f'''(x_i)}{3!}(x_{i-1} - x_i)^3 + \frac{f^{(4)}(x_i)}{4!}(x_{i-1} - x_i)^4 + \frac{f^{(5)}(x_i)}{5!}(x_{i-1} - x_i)^5 + \dots$$

$$f'(x_i) = \frac{f(x_{i-1}) - f(x_i)}{(x_{i-1} - x_i)} - \left(\frac{f''(x_i)}{2!}(x_{i-1} - x_i) + \frac{f'''(x_i)}{3!}(x_{i-1} - x_i)^2 + \frac{f^{(4)}(x_i)}{4!}(x_{i-1} - x_i)^3 + \frac{f^{(5)}(x_i)}{5!}(x_{i-1} - x_i)^4 + \dots \right)$$

$$f'(x_i) \cong \frac{f(x_{i-1}) - f(x_i)}{(x_{i-1} - x_i)}$$

$$f(x_{i+1}) = \sum_{n=0}^{\infty} \frac{f^{(n)}(x_i)}{n!} (x_{i+1} - x_i)^n \qquad f(x_{i-1}) = \sum_{n=0}^{\infty} \frac{f^{(n)}(x_i)}{n!} (x_{i-1} - x_i)^n$$

$$f(x_{i+1}) = f(x_i) + f'(x_i)(x_{i+1} - x_i) + \frac{f''(x_i)}{2!}(x_{i+1} - x_i)^2 + \frac{f'''(x_i)}{3!}(x_{i+1} - x_i)^3 + \frac{f^{(4)}(x_i)}{4!}(x_{i+1} - x_i)^4 + \frac{f^{(5)}(x_i)}{5!}(x_{i+1} - x_i)^5 + \dots$$

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$$f(x_{i+1}) - f(x_{i-1}) = f'(x_i)((x_{i+1} - x_i) - (x_{i-1} - x_i)) + \frac{f''(x_i)}{2!}((x_{i+1} - x_i)^2 - (x_{i-1} - x_i)^2) + \frac{f'''(x_i)}{3!}((x_{i+1} - x_i)^3 - (x_{i-1} - x_i)^3) + \dots$$

$$f(x_{i+1}) - f(x_{i-1}) = f'(x_i)(x_{i+1} - x_{i-1}) + \frac{f''(x_i)}{2!}((x_{i+1} - x_i)^2 - (x_{i-1} - x_i)^2) + \frac{f'''(x_i)}{3!}((x_{i+1} - x_i)^3 - (x_{i-1} - x_i)^3) + \dots$$

$$f'(x_i) \cong \frac{f(x_{i+1}) - f(x_{i-1})}{(x_{i+1} - x_{i-1})}$$

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$$f'(x_i) \cong \frac{f(x_i + h) - f(x_i)}{h}$$

Si los datos se encuentran equiespaciados podemos utilizar las expresiones que analizamos anteriormente

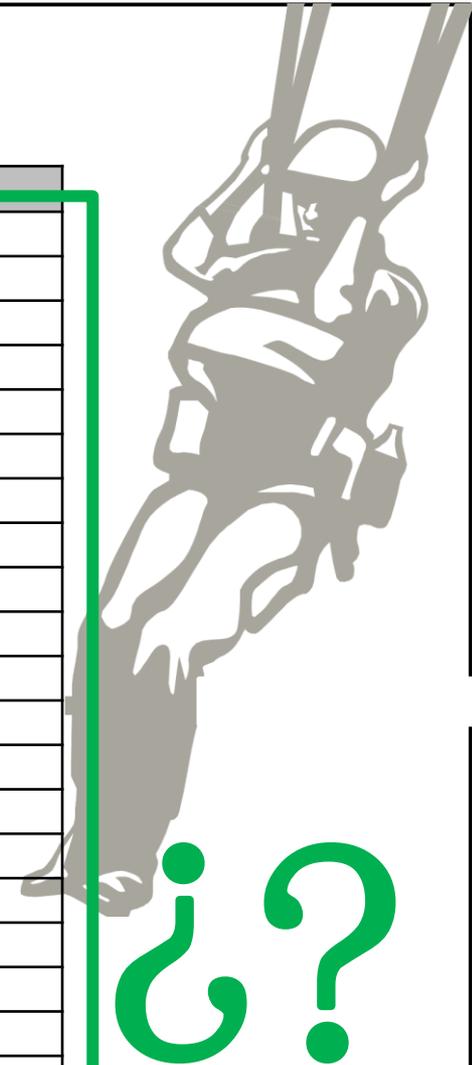
Un paracaidista se arroja de un avión y mediante un dispositivo de medición se almacena su ubicación respecto del suelo con un intervalo de tiempo de dos segundos. Luego de procesar los datos se obtiene la siguiente tabla:

Tiempo (seg)	Distancia (m)
2	17.41
4	62.27
6	126.17
8	203.24
10	289.44
12	381.96
14	478.86
16	578.80
18	680.84
20	784.34
22	888.85
24	994.05
26	1099.74
28	1205.76
30	1312.02
32	1418.44
34	1524.97
36	1631.58
38	1738.24
40	1844.93



Averiguar su velocidad y aceleración instantánea

Tiempo (seg)	Distancia (m)	Velocidad (m/seg)	Aceleración(m/seg ²)
2	17.41		
4	62.27		
6	126.17		
8	203.24		
10	289.44		
12	381.96		
14	478.86		
16	578.80		
18	680.84		
20	784.34		
22	888.85		
24	994.05		
26	1099.74		
28	1205.76		
30	1312.02		
32	1418.44		
34	1524.97		
36	1631.58		
38	1738.24		
40	1844.93		



Velocidad vs Tiempo

